3.2 CURVED MEMBERS

When linear members are curved, the analysis of the member is no longer as simple as it is for a straight member. The stability considerations change from those in a straight member. The stresses at a location diverge from that obtained from normal flexural theory. Furthermore, additional stresses occur that can be more of a problem than the flexural stresses. The flexibility of curved members also tends to increase with increase in curvature.

The first section will cover the member in a moderately curved situation where overall member behavior is the major concern. The second section will cover the member cross section and how it behaves when curved, especially when highly curved.

3.2.1 EFFECT ON FLEXURAL AND COMPRESSION BEHAVIOR

A curved member, if restrained from relative lateral translation of the ends, can exhibit completely different in-plane behavior than a straight member or a curved member with end movement. Under compression induced by lateral loads, such a member will buckle as shown in Figure 3.2.1-1.

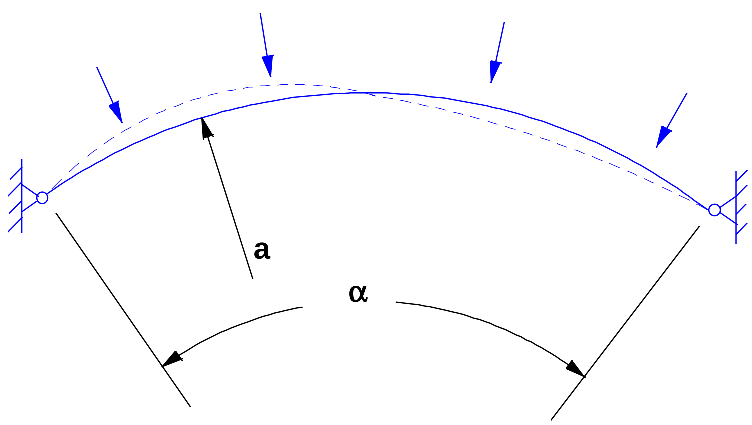

Figure 3.2.1-1 Buckling of curved compression members

The effective length is approximately half of the arc length. The effective length can be calculated as:

$$\frac{L_{eff}}{L} = \frac{\pi}{\sqrt{4\pi^2 - \alpha^2}}; \; \alpha \leq \pi$$ **Equation 3.2.1-1**

where α = included angle in radians

If the ends are rotationally restrained, the effective length will be even lower. Equation 3.2.1-1 can still be used if the inflection point location near the ends can be determined or estimated. Measure to the inflection point in this case.

Often a tie member is employed to keep the ends from translating. This is especially important when the curved member becomes very shallow and requires large thrust with little end displacement to carry the load.

When the rise-to-span ratio (see Figure 3.2.1-2) is less than about 0.07, the member should be analyzed together with its tie system to evaluate the end displacement or the member should be considered only as a flexural member under lateral loads.

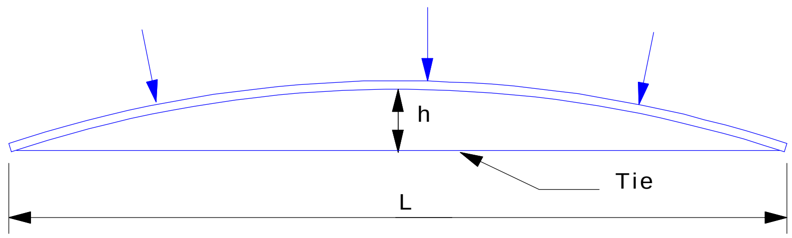

Figure 3.2.1-2 Curved compression member with tie

If a curved member is subjected to end loads as shown in Figure 3.2.1-3 and the ends can deflect, the situation is that of a column with initial eccentricity.

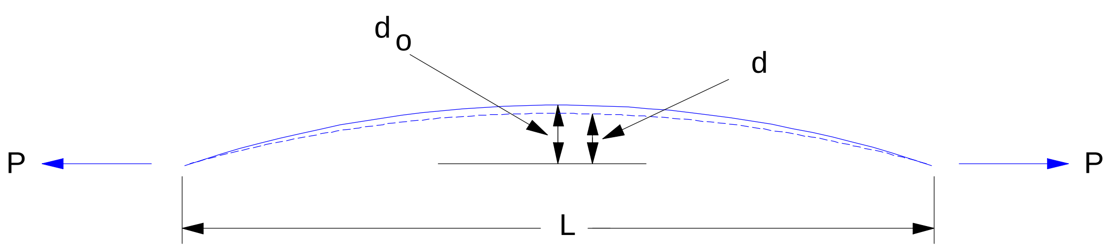

Figure 3.2.1-3 Curved member with end loading

If the value of d_o is small relative to its length, the final lateral position d and the corresponding maximum stress f_{max} can be evaluated as follows:

$$d = \frac{d_o}{\left(1 + \frac{P}{P_e}\right)}$$ **Equation 3.2.1-2**

$$f_{max} = \frac{P}{A}\left(1 + \frac{d_o c}{r}\,\frac{P_e}{P + P_e}\right)$$ **Equation 3.2.1-3**

where A = cross sectional area
r = radius of gyration
c = distance to extreme fiber
P_e = Euler load from Equation 3.2.1-4
P = applied load; tension positive, compression negative

$$P_e = \frac{\pi^2 EA}{\left(\frac{L_{eff}}{r}\right)^2}$$ **Equation 3.2.1-4**

The stress f_{max} can be compared with the axial stress limit or the axial and flexural stress can be computed separately and used in the interaction equations in Section 3.1.3.7.

Considering the curved member in Figure 3.2.1-3 as a single straight member for purposes of computer modeling is acceptable for $\mathbf{d_o / L} < .07$. When this is done the axial stiffness is reduced. An effective area can be estimated for modeling purposes as:

$$A_{eff} = \frac{A}{1 + 0.52\left(\frac{d_o}{r}\right)^2}$$ **Equation 3.2.1-5**

where **r** is the radius of gyration of the section.

For larger curvatures the curved member would best be considered as a series of straight line or shallow curved segments.

If a curved member is subjected to uniform moment, its lateral stability can be larger or smaller than for a straight member of the same length in which bending occurs about the strong axis. Timoshenko[1] illustrates this for sections with St. Venant torsional stiffness, but with no warping stiffness. Flexure producing compression on the outside of the bend produces a buckling moment less than that of a straight bar while compression on the inside of the bend leads to a higher buckling moment than in a straight bar.

For curved members subject to lateral buckling (especially if the outside is in compression) it is very important to restrain the ends of the curved member against weak axis rotation in order to achieve significant lateral stability. Significant improvement of lateral stability can be achieved by intermediate lateral bracing. Examination of the member should also include a careful evaluation of the integrity of the cross section under flexure, as covered in the next section.

3.2.2 BEHAVIOR OF SECTIONS AT LOCATIONS OF HIGH CURVATURE

Behavior of sections that are not straight or mildly curved diverge from normal flexural theory in several aspects.

3.2.2.1 Sections with Webs in Plane of Curvature ....

The inner fibers of a curved section have flexural stresses higher than obtained from normal flexural theory. Radial stresses are developed in the webs from the bending moments. The portions of flanges away from the webs lose their effectiveness and transverse bending occurs.

When **a/c**, the ratio of radius of the curved beam to the distance from inner fiber to centroidal axis (see Figure 3.2.2.1-1) is less than 10, the stress increase at the inner fiber should be considered. A corresponding decrease from normal flexural theory occurs at the outer fiber of the section. At **a/c** of 10 the stress increase is about 7 percent.

The stress in a curved member can be calculated using the Winkler-Bach formula:

$$f_b = \frac{M}{Aa}\left[1 + \frac{y}{Z(a+y)}\right]$$ **Equation 3.2.2.1-1**

where A = cross sectional area of the section
y = distance from the centroidal axis (+y away from center of curvature)
Z = section property defined in Equation 3.2.2.1-2 to Equation 3.2.2.1-4

The neutral axis moves toward the center of curvature as shown in Figure 3.2.2.1-1. Its location can be determined directly from Equation 3.2.2.1-1.

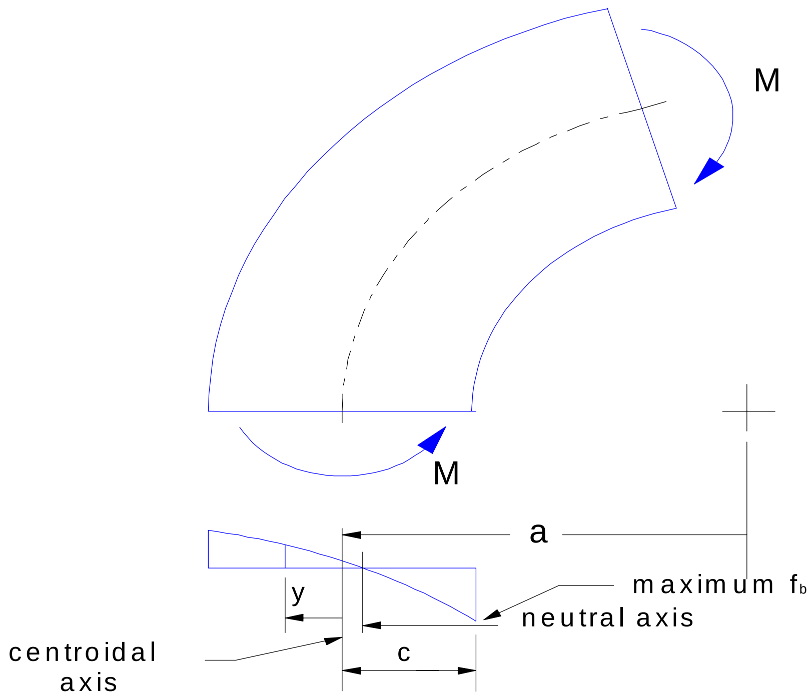

Figure 3.2.2.1-1 Flexural stress in a curved member

$$Z = -1 + \frac{a}{h} \ln \frac{a+c}{a-c}$$

Equation 3.2.2.1-2

where a, c, and h are defined in Figure 3.2.2.1-2(a)

$$Z = -1 + \frac{a}{A}\left[b \ln \frac{a_o}{a_i} - (b - t_w) \ln \frac{a_o - t}{a_i + t} \right]$$

Equation 3.2.2.1-3

where a, a_i, a_o, b, t, and t_w are defined in Figure 3.2.2.1-2(b)

$$Z = -1 + \frac{a}{A}\left[(b'_o + t_w) \ln a_o - (b'_i + t_w) \ln a_i - b'_o \ln(a_o - t) + b'_i \ln(a_i + t) \right]$$

Equation 3.2.2.1-4

where a, a_i, a_o, b'_i, b'_o, c, t, and t_w are defined in Figure 3.2.2.1-2(c)

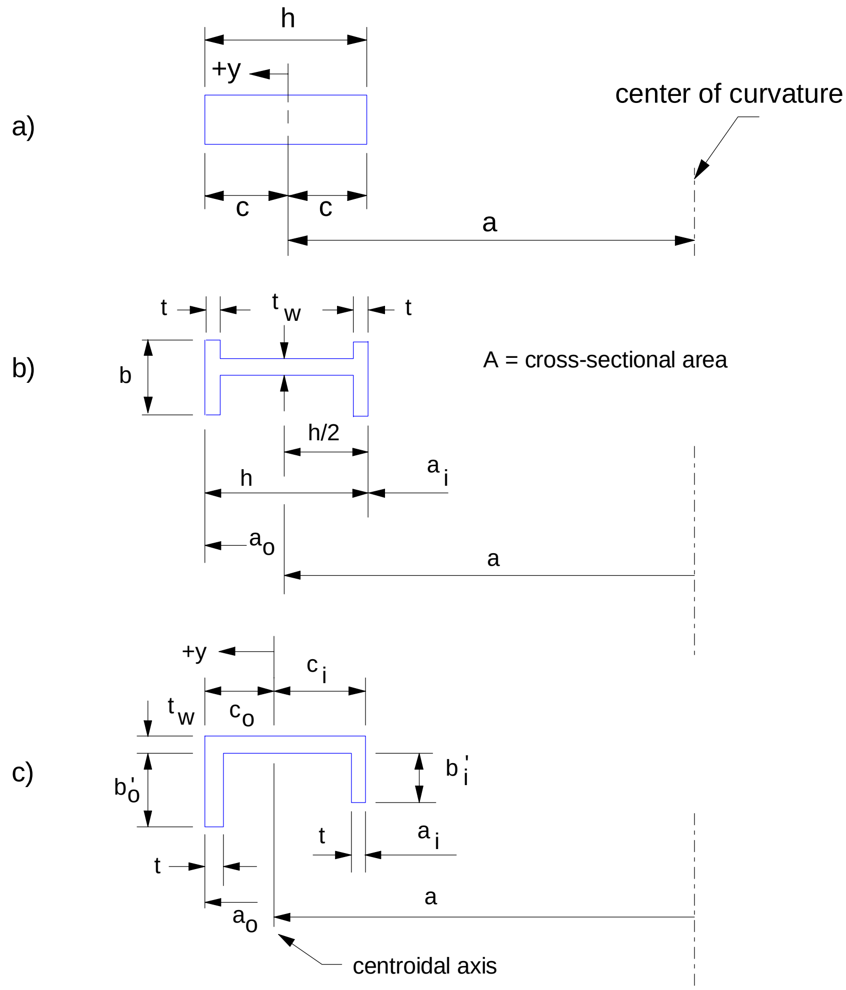

Figure 3.2.2.1-2 Section property Z for curved members

When the section has a flange, the flexural stress in the curved flange distorts the flange. Tension pulls the flange inward and compression pushes the flange outward away from the webs as illustrated in Figure 3.2.2.1-3 for **I** and rectangular tube sections.

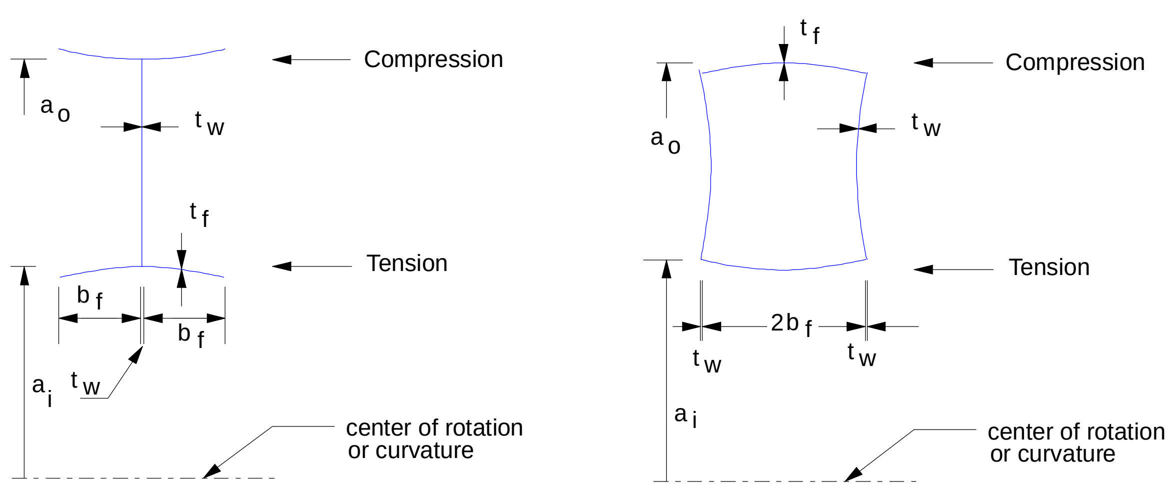

Figure 3.2.2.1-3 Flange distortion in curved members

As a result, the tips of the flanges or the region of the flange away from the webs relieves itself of stress.

This has led to the use of an effective width concept[2] much like that used for the stiffened compression flange. However, in this case, the effective width is independent of stress level and the concept works well for relatively thin flanges as well as for thick flanges. This effective width can be approximated simply by the following relationship:

$$b' = 0.7\sqrt{a_f t_f} \leq b_f$$

Equation 3.2.2.1-5

where b' = effective flange width adjacent to web
a_f = radius of curvature at flange
t_f = flange thickness
b_f = flange width beyond web as shown in Figure 3.2.2.1-3

The lesser of the effective width for compression flanges per Section 3.1.2.1 and **b′** should be used. The **b′** value is likely to control for thin wall sections. The centroidal axis and area should be evaluated using the effective section with **Z** being computed using Equation 3.2.2.1-4 since the flange widths are likely to be unequal.

It is recommended to minimize the bend radius at the corners for maximum stiffness. For purposes of analysis, ignore the bend radii and measure **b′** from the face of the web. The standard moment of inertia of the section considering the **b′** effective widths as shown in Figure 3.2.2.1-2 is quite satisfactory for stiffness computations. The correct **I** is never more than 8 percent larger than the standard **I**.

Transverse bending stress in the flanges is likely to be larger than the circumferential stress evaluated by Equation 3.2.2.1-1. This bending stress induced from the flange distortion illustrated in Figure 3.2.2.1-3 can be calculated approximately from the following expression:

$$f_{btr} = Bf_b$$

Equation 3.2.2.1-6

$$\text{where } B = 17\frac{b_f}{\sqrt{a_f t_f}} \leq 17$$

Equation 3.2.2.1-7

The $\mathbf{f_b}$ used should be the stress at the center of the flange ($t_f/2$ from face), but the outer fiber stress is acceptable for thin-flanged sections.

The radial stress in the web can also be significant. It is critical in the web at the junction with the flange. This stress as illustrated in Figure 3.2.2.1-4 can be approximated from hoop tension relationships to be:

$$f_r = \frac{F_f}{a_f t_w}$$

Equation 3.2.2.1-8

The numerator, $\mathbf{F_f}$, is the flange force so the stress at the center of the flange is the proper value to use. This force, if compressive, in conjunction with compressive circumferential stresses in the web can lead to web crippling.

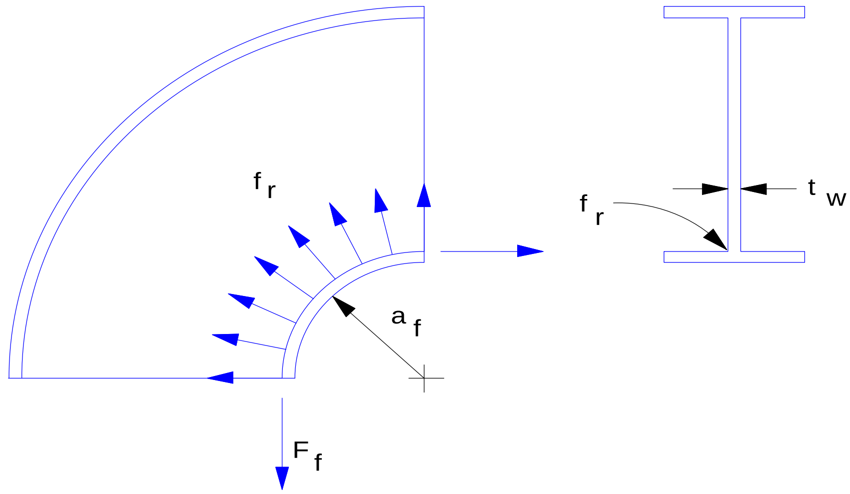

Figure 3.2.2.1-4 Radial web stress in curved members

Example 3.2-1 in Section 6.2.2.1 illustrates the evaluation of section properties and stresses in a curved member with web(s) in the plane of curvature.

3.2.2.2 Curved Circular Tubular Members

When a circular member which is curved is subject to flexure, it distorts in a manner illustrated in Figure 3.2.2.2-1.

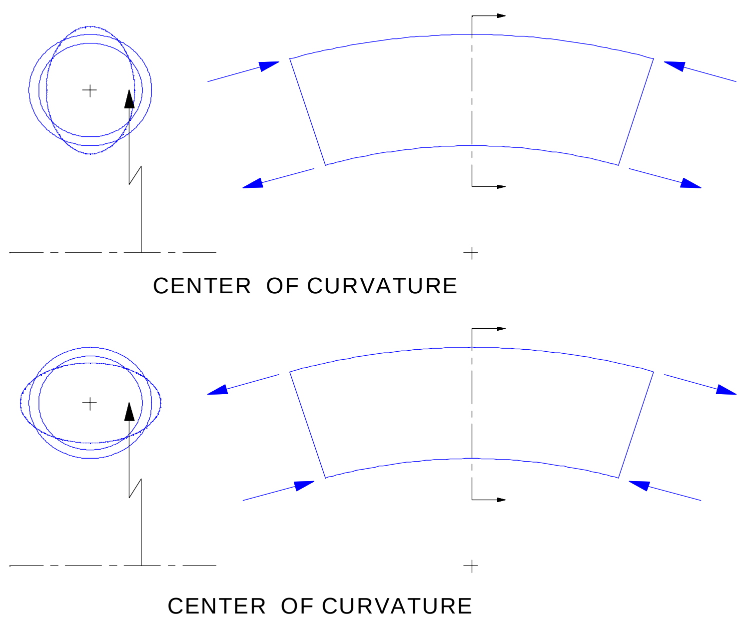

Figure 3.2.2.2-1 Distortion in curved circular members

When the inner portion is in tension and the outer portion in compression, the section will ovalize and elongate in the plane of bending. When the stresses are reversed, the section will flatten.

This behavior increases the flexibility and the stresses beyond that calculated from normal flexural theory. The parameter that defines the situation is:

$$at/R^2$$

where a = radius of curvature to centroidal axis
t = wall thickness
R = pipe radius

This parameter is called the flexibility characteristic, **g**, in piping design.

With the flexibility characteristic, the following expressions from von Karman[3,4,5,] can be used to modify the tube moment of inertia in order to obtain a proper flexibility.

$$\text{effective } I = jI$$

Equation 3.2.2.2-1

$$\text{where } j = 1 - \frac{9}{10 + 12g^2};\ g \geq 0.335$$

Equation 3.2.2.2-2

To obtain a value for the maximum stress from flexure, a stress intensification factor, **i**, can be used to modify the standard stress calculated.

$$f_{bmax} = \frac{iMR}{I}$$

Equation 3.2.2.2-3

$$\text{if } g < 1472; \quad i = \frac{2}{3j\sqrt{3q}} \geq 1$$

$$\text{if } g \geq 1472; \quad i = \frac{1-q}{j}$$

Equation 3.2.2.2-4

$$\text{where } q = \frac{6}{5 + 6g^2}$$

Equation 3.2.2.2-5

A somewhat simpler set of expressions for **i** and **j** are used in pipe stress analysis work.

$$j = \frac{g}{165} \leq 1$$

Equation 3.2.2.2-6

$$i = \frac{0.9}{g^{2/3}} \geq 1$$

Equation 3.2.2.2-7

The use of both these sets of expressions should be limited to cases in which **a/R** is greater than or equal to 2. The agreement between the two approaches is reasonably good for low levels of **g** used in Example 3.2-2 in Section 6.2.2.2. The values of **j** are reasonably close until **g** > 1.2 (see **Figure 3.2.2.2-2**). The value of **i** drops below 1 for **g** > 0.73 using Equation 3.2.2.2-4 while **i** reaches 1 at **g** = 0.854 in Equation 3.2.2.2-7 (see Figure **3.2.2.2-3**).

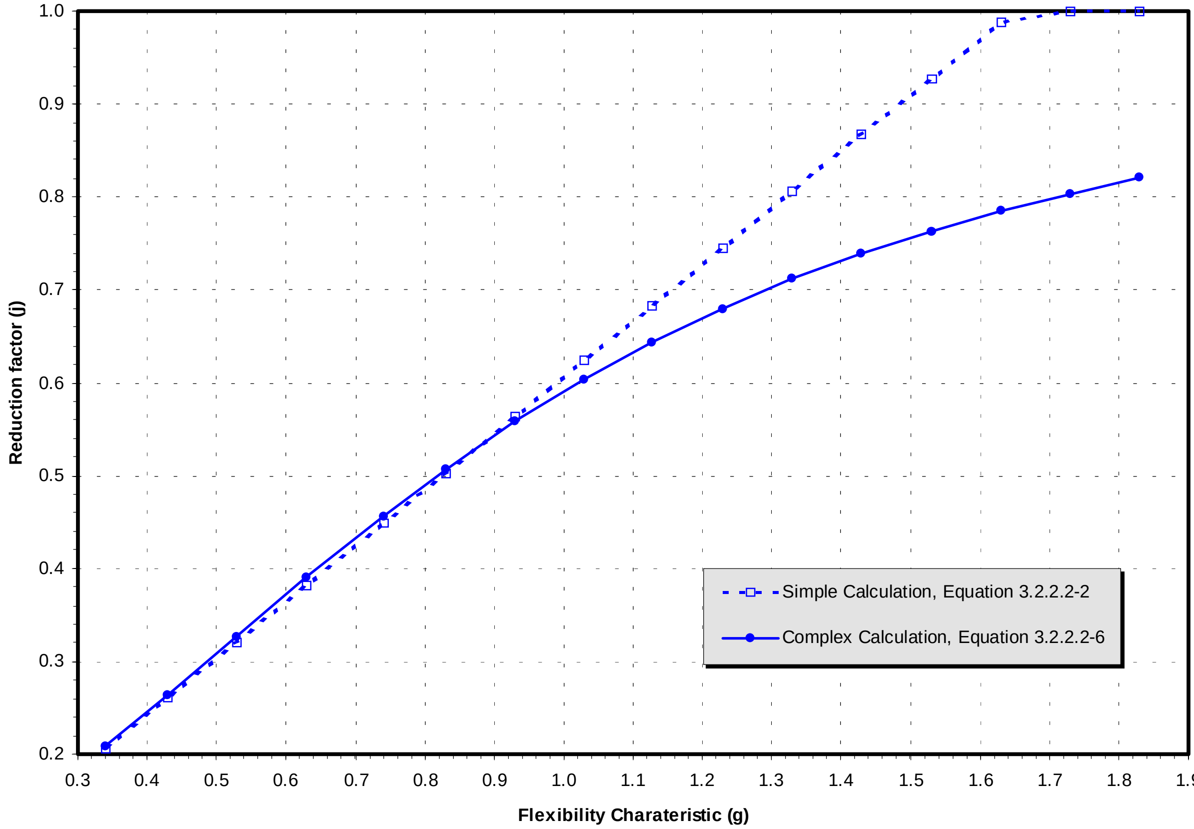

Figure 3.2.2.2-2 Inertia reduction factor (j)

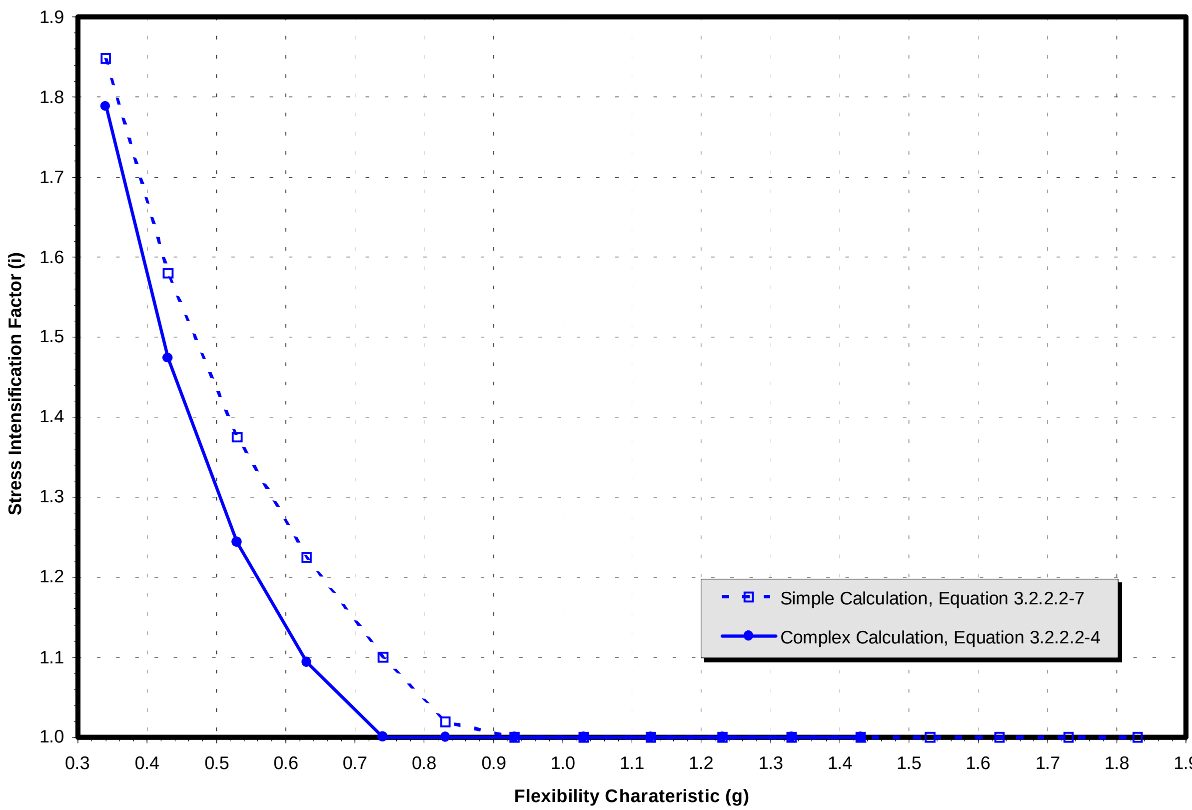

Figure 3.2.2.2-3 Stress intensification factor (i)

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