

	A	B	C	D	E	F	G	H	I
1	Select Data set:	Sedan							
2									
3	Subsystem	Subsystem Mass Fraction of Curb Mass	User Defined Mass (optional)	Preliminary Subsystem Mass (kg)	Preliminary Mass Estimate (kg)				
4	Body Non-structure	0.204		305.70					
5	Body Structure	0.227		340.17					
6	Front Suspension	0.049		73.43					
7	Rear Suspension	0.044		65.94					
8	Braking	0.032		47.95					
9	Powertrain	0.185		277.23					
10	Fuel & Exhaust	0.040		59.94					
11	Steering	0.014		20.98					
12	Tires & Wheels	0.065		97.41					
13	Electrical	0.046		68.93					
14	Cooling	0.027		40.46					
15	Bumpers	0.022		32.97					
16	Closures	0.045		67.43					
17									
18	Vehicle Plan View Area (m2)		8.58858						
19	Vehicle Curb Mass (kg)			1498.55					
20									
21	Number of Passengers		5	350.00					
22	Cargo Mass (kg)			120.00					
23									
24	Gross Vehicle Mass			1968.55					

Preliminary Vehicle Mass Estimation Using Empirical Subsystem Influence Coefficients



www.a-sp.org

Report on

Preliminary Vehicle Mass Estimation Using Empirical Subsystem Influence Coefficients

by

Donald E Malen, *Adjunct Assistant Professor*
Kundan Reddy, *Masters Candidate*

*InterProfessional Programs
University of Michigan
Ann Arbor, MI*

Prepared for
The FGPC-Mass Compounding Project Team
Auto/Steel Partnership
2000 Town Center, Suite 320
Southfield, Michigan

May 9, 2007
(Revised June 26, 2007)

Acknowledgments

The authors deeply appreciate the guidance and support provided by the *FGPC-Mass Compounding* project team:

J. R. Shaw, US Steel Corp. (Chairman); H. Ezzat , T. M. Lee, C. C. Leung, J. Polewarczyk, General Motors Corp., V. C. Shah, DaimlerChrysler Corp.; R. Sohmshtetty, S. Das, Ford Motor Co.; C. Potter, A.I.S.I.; and R. A. Heimbuch, P. J Villano, Auto/Steel Partnership.

Special appreciation is due to the diligent efforts of C. Leung and V. Shah in providing vehicle mass data and their patience in the several iterations of consolidating the data. The authors also appreciate the data provided by L. Pecar of the GM Benchmarking Activity.

Executive Summary

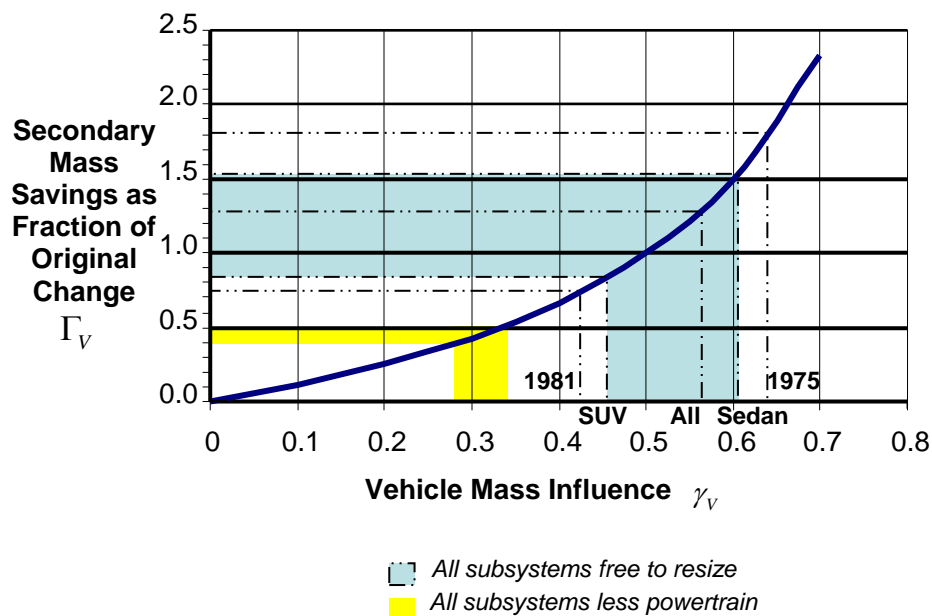
Vehicle design engineers intuitively know that an unplanned mass increase in a component during vehicle design has a ripple effect throughout the vehicle; other components need to be resized increasing vehicle mass even more. The phrase *mass begets mass* describes this phenomenon. A more encouraging view of this behavior is considering a reduction in the mass of a component enabled by a new technology resulting in a greater mass saving for the overall vehicle. These secondary mass changes can be considerable—estimated at an additional 0.7 to 1.8 times the initial mass change.

This mass compounding behavior may be modeled using subsystem mass influence coefficients—the incremental change in subsystem mass for a unit change in gross vehicle mass. Published data on influence coefficients is sparse, and that published are based on vehicles in the 1975–1981 model years. The purpose of this study is to update influence coefficients using contemporary vehicles.

In this study, mass data from 35 vehicles have been analyzed to determine mass influence coefficients. The vehicles in the study covered Sedans, SUVs, Pick Ups, and Vans (See Appendix C for the specific models used). Linear regression was used to determine the influence coefficients.

The primary results of this study are summarized in the illustration below. The secondary mass change is that additional change due to resizing components when a vehicle mass change occurs during design (the primary change). This secondary change depends on the vehicle mass influence coefficient—the sum of coefficients for the subsystems. Indicated are results of previous studies from 1975 and 1981, along with results of this study for *All* vehicles, the *SUV* group, and the *Sedan* group. (The *Pick Up* and *Van* categories had an insufficient number of vehicles to be represented alone.)

When all subsystems can be resized, the secondary mass savings is from 0.8 to 1.5 kg/kg (1.25kg/kg is the estimate for the *All* vehicles group). When the powertrain has been fixed and is not available for resizing, the secondary mass savings is from 0.4 to 0.5 kg/kg (0.5kg/kg is the estimate for the *All* vehicles group).



Section 8 provides an in depth example application of this data.

Table of Contents

1. Acknowledgements	2
2. Executive Summary	3
3. Preliminary Mass Estimation in Vehicle Design	5
4. Mass Compounding Model	5
5. Previous work on Mass Influence Coefficients	6
6. Description of this study	7
7. Summary of Results	8
8. Example Applications	9
9. Bibliography	14

Appendices

Technical Appendix A Derivation of mass model with influence coefficients	15
--	-----------

Technical Appendix B Estimation of subsystem influence coefficients	18
--	-----------

Data Appendix C

Vehicles used in analysis	22
---------------------------	----

Functional Subsystem Mass Categories	23
--------------------------------------	----

- *Gross Vehicle Mass vs. Plan View Area*
- *Curb Mass vs. Plan View Area*
- *System Mass as a percentage of Curb Mass*
- *Summary of Influence Coefficients*
- *Subsystem Mass vs. Gross Vehicle Mass-Linear*
- *Subsystem Mass vs. Gross Vehicle Mass-Log*
repeated for

1) All Vehicles	24
2) Sedans	29
3) SUVs	34
4) Pick Ups	39

Tabular Summary of Subsystem Influence Coefficients	44
---	----

Selection of Influence Coefficients for Inclusion in Model	45
--	----

3. Preliminary Mass Estimation in Vehicle Design

Vehicle mass is an important parameter during design. Subsystems are sized based on vehicle mass, and important product characteristics such as acceleration performance and fuel economy depend on vehicle mass. Given this importance of mass in vehicle design, mass estimation becomes a critical activity. The purpose of mass estimation varies with the design stage.

During the product planning stage, or *pre-configuration stage*, the purpose of mass estimation is to determine general feasibility of a program and to set initial mass goals for vehicle subsystems. At this stage, only a broad vehicle mission is identified; for example vehicle type and number of passengers. Because of this sparse data, mass estimation is often based on first order statistical models based on contemporary vehicles.

In the next stage of design, the *configuration stage*, the purpose of mass estimation is to aid in decision making about which alternative technologies to include in the program, and also to add additional precision to subsystem mass goals. In this stage, mass estimation is based on both semi-empirical models and statistical data.

During the detail design stage, *post-configuration stage*, the goal of mass estimation is monitor mass growth and to control to target. As the vehicle configuration is set, mass estimation is based on actual subsystem designs and prototypes.

In this paper, we focus on the *pre-configuration* and *configuration* stages. These are particularly significant as lasting decisions are made about which technologies and subsystem will be used. The implications of these decisions shadow all subsequent work.

4. Mass Compounding Model

Vehicle design engineers intuitively know that an unplanned mass increase in a component during vehicle design has a ripple effect throughout the vehicle; other components need to be resized increasing vehicle mass even more. The phrase *mass begets mass* describes this phenomenon. A more encouraging view of this behavior is considering a reduction in the mass of a component enabled by a new technology resulting in a greater mass saving for the overall vehicle. These secondary mass changes can be considerable—estimated at an additional 0.7 to 1.8 times the initial—primary--mass change.

A means to quantify the secondary mass change is with a *mass compounding model*. In this model, each subsystem (denoted by i) is assigned an *influence coefficient*, γ_i . The influence coefficient is the change in the subsystem mass when gross vehicle mass undergoes a unit change. The physical interpretation of the influence coefficient is each subsystem is sized to some degree by the mass of the vehicle, and as the vehicle mass changes the subsystem must also be resized.

The typical question we are interested in answering is;

Given a balanced vehicle under design, a primary mass change now occurs during design. What is the final vehicle mass after resizing subsystems?

The answer depends on the vehicle mass influence coefficient, γ_{VEH} . This the sum of coefficients for all subsystems which may undergo resizing. The resulting vehicle mass is given by Equation 4.1 (see Appendix A for a derivation).

$$W_{\infty} = W_0 + \Delta + \Delta\Gamma_v \quad \text{Equation 4.1}$$

$$\Gamma_v = \left[\frac{\gamma_v}{(1 - \gamma_v)} \right] \quad \text{Secondary Mass Coefficient for Vehicle} \quad \text{Equation 4.2}$$

where W_0 is the initial vehicle mass for which the subsystems are sized
 Δ is the initial mass change (primary mass change)
 W_{∞} is the resized, compounded vehicle mass
 $\Delta\Gamma_v$ is the additional (secondary) mass change
 γ_v is the mass influence coefficient for the vehicle given by $\gamma_v = \sum_{i=1}^n \gamma_i$
 γ_i is the mass influence coefficient for subsystem i

The resulting mass for subsystem i due to an initial increase of Δ ;

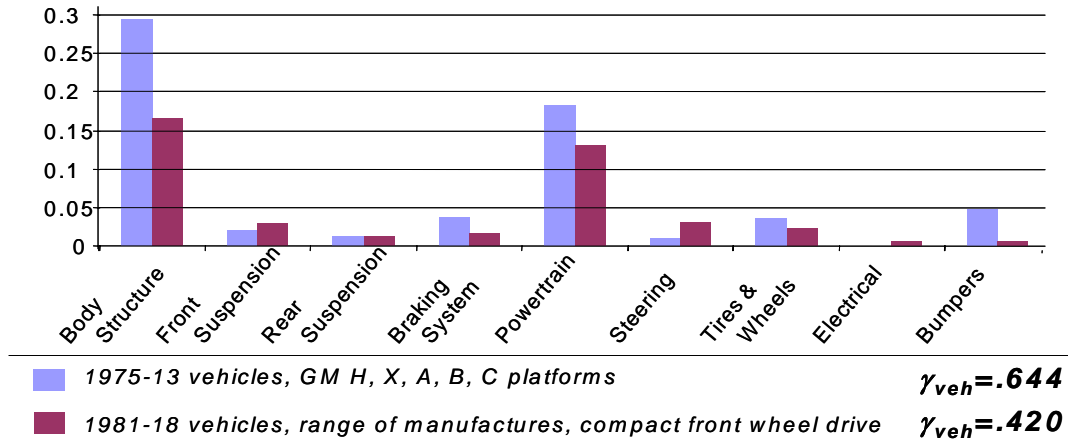
$$w_{i\infty} = w_i + \Delta\Gamma_i \quad \text{Equation 4.3}$$

$$\Gamma_i = \left[\frac{\gamma_i}{(1 - \gamma_v)} \right] \quad \text{Secondary Mass Coefficient for Subsystem i} \quad \text{Equation 4.4}$$

where w_i is the initial subsystem mass
 $w_{i\infty}$ is the resized subsystem mass
 $\Delta\Gamma_i$ is the additional secondary change
 γ_v, γ_i and Δ are given above

5. Previous work on Mass Influence Coefficients

Published data on influence coefficients is sparse, and those published are based on vehicles in the 1975–1981 model years [1, 2, 3]. Figure 5.1 summarizes the subsystem mass influence coefficients for two of these studies. Both studies used linear regression of measured subsystem mass data as described in the next section.



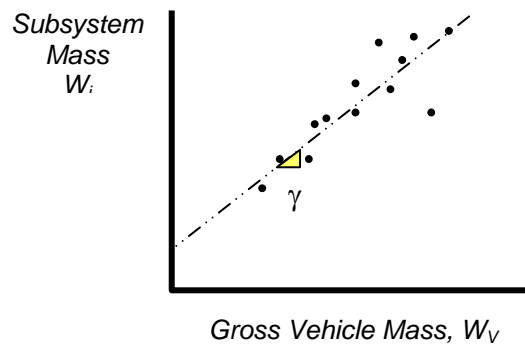
Summary of Previous Mass Influence Coefficients

Figure 5.1

Based on these influence coefficients, the secondary vehicle mass savings using Equation 4.2 is 1.81 for the 1975 study, and 0.72 for the 1981 study. Note that resizing the body structure and powertrain contribute the most to this secondary mass savings.

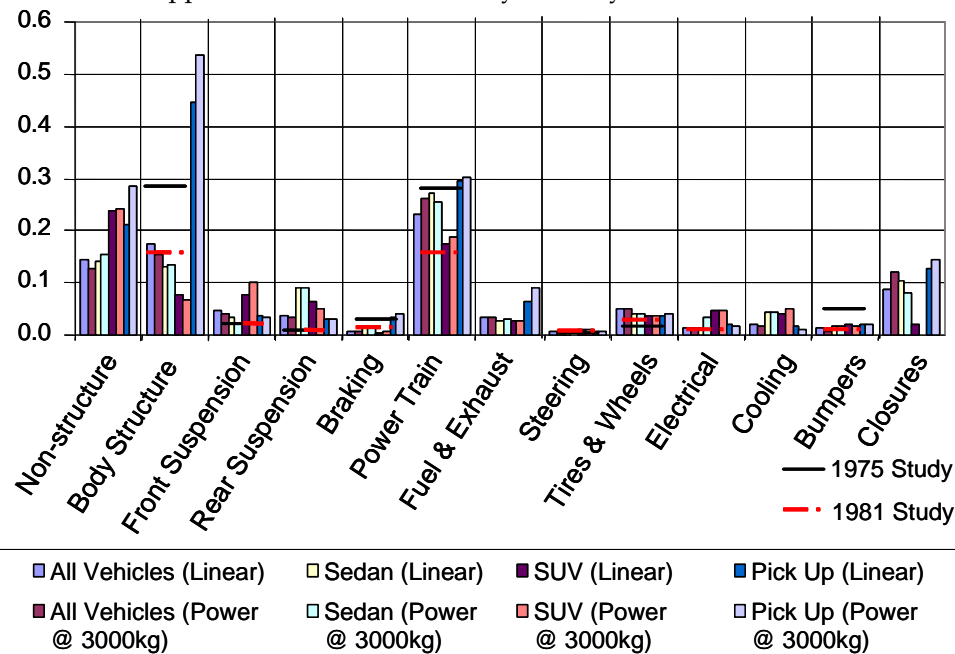
6. Description of this study

In this study, we identify subsystem influence coefficients using regression. A function is fit relating the mass of subsystem i to gross vehicle mass, Figure 6.1. The influence coefficient is the slope of the fit curve. Data from 32 vehicles were used for the fit. The 32 vehicles were further divided into Sedan, SUV, and Pick up groups. Appendix B describes the model and the method used for estimation. Appendix C shows the scatter plots for each subsystem for each vehicle group.



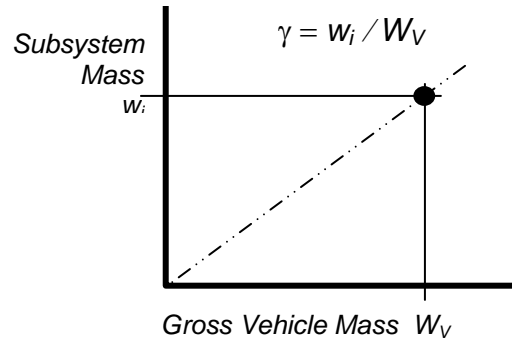
Influence Coefficient By Linear Regression
Figure 6.1

Figure 6.2 shows a comparison of influence coefficients by subsystem. Shown are the results for each vehicle group for both a linear function and also a power function, Equation B.3. A more complete table of results is contained in Appendix C-Tabular Summary of Subsystem Influence Coefficients.



Subsystem Mass Influence Coefficients Determined in this Study
Figure 6.2

An alternative means to estimate the mass influence coefficient is by the Ratio Method [7]. In this method, only the single reference vehicle data is needed. The subsystem mass-to-gross vehicle mass relationship is assumed as shown in Figure 6.3. In this case, the influence coefficient is given by Equation 5.1.



Influence Coefficient By Ratio
Figure 6.3

Ratio method for influences coefficient determination,

$$\gamma_i = \frac{w_i}{W_v} \quad \text{Equation 6.1}$$

where w_i current subsystem mass
 W_V original gross vehicle mass

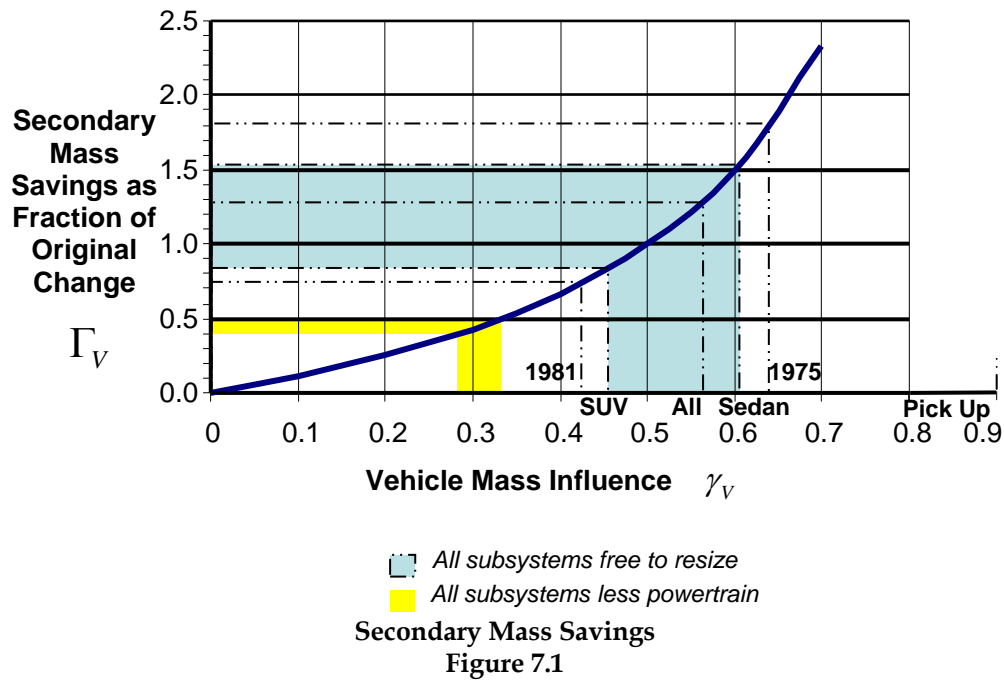
7. Summary of Results

The results of this study are summarized in Table 7.1 and graphically in Figure 7.1.

	All	Sedan	SUV	Ratio (sedan)
Vehicle Influence Coefficient <i>All subsystems free to resize</i>	.562	.599	.463	.532
Vehicle Influence Coefficient <i>All subsystems free to resize Except Powertrain</i>	.332	.331	.290	.391
Secondary Mass savings <i>All subsystems free to resize</i>	1.28 Times primary mass change	1.49 times	0.86 times	1.14 times
Secondary Mass Savings <i>All subsystems free to resize Except Powertrain</i>	0.50 times	0.49 times	0.41 times	0.64 times

Secondary Mass Savings
Table 7.1

Indicated on Figure 7.1 are results of previous studies from 1975 and 1981, along with results of this study for *All* vehicles, the *SUV* group, and the *Sedan* group. (The *Pick Up* and *Van* categories had an insufficient number of vehicles to be represented alone.) When all subsystems can be resized, the secondary mass savings is from 0.8 to 1.5 kg/kg (1.25kg/kg is the estimate for the *All* vehicles group). When the powertrain is not available for resizing, the secondary mass savings is from 0.4 to 0.5 kg/kg (0.5kg/kg is the estimate for the *All* vehicles group).



8. Example Application

A new vehicle is in the planning stage (*pre-configuration*). It is targeted at 5 passengers with a 120kg cargo capacity. The target test weight class for fuel economy evaluation is 2250Lb (1950Lb or 886kg curb mass). Mass reduction is a primary goal and several technologies are under investigation for application. The objectives of mass analysis at this stage are to

- 1) determine an initial mass estimate for the vehicle based on conventional design
- 2) determine the most effective mass reduction technologies to apply
- 3) estimate the vehicle mass when these technologies are used
- 4) set subsystem mass goals for the detail design stage which are consistent with 1) , 2), & 3).

Step 1: Pre Configuration Mass Estimation

From human accommodation and dimensional benchmarking, the vehicle length is estimated at 4.732 m and width at 1.815m. From this sparse data, the vehicle curb mass can be estimated given the plan view area (relationship in Appendix C, Sedan section).

$$\text{curb mass (kg)} = 147.75A(m^2) + 229.59$$

$$A = (1.815m)(4.732m) = 8.58858m^2$$

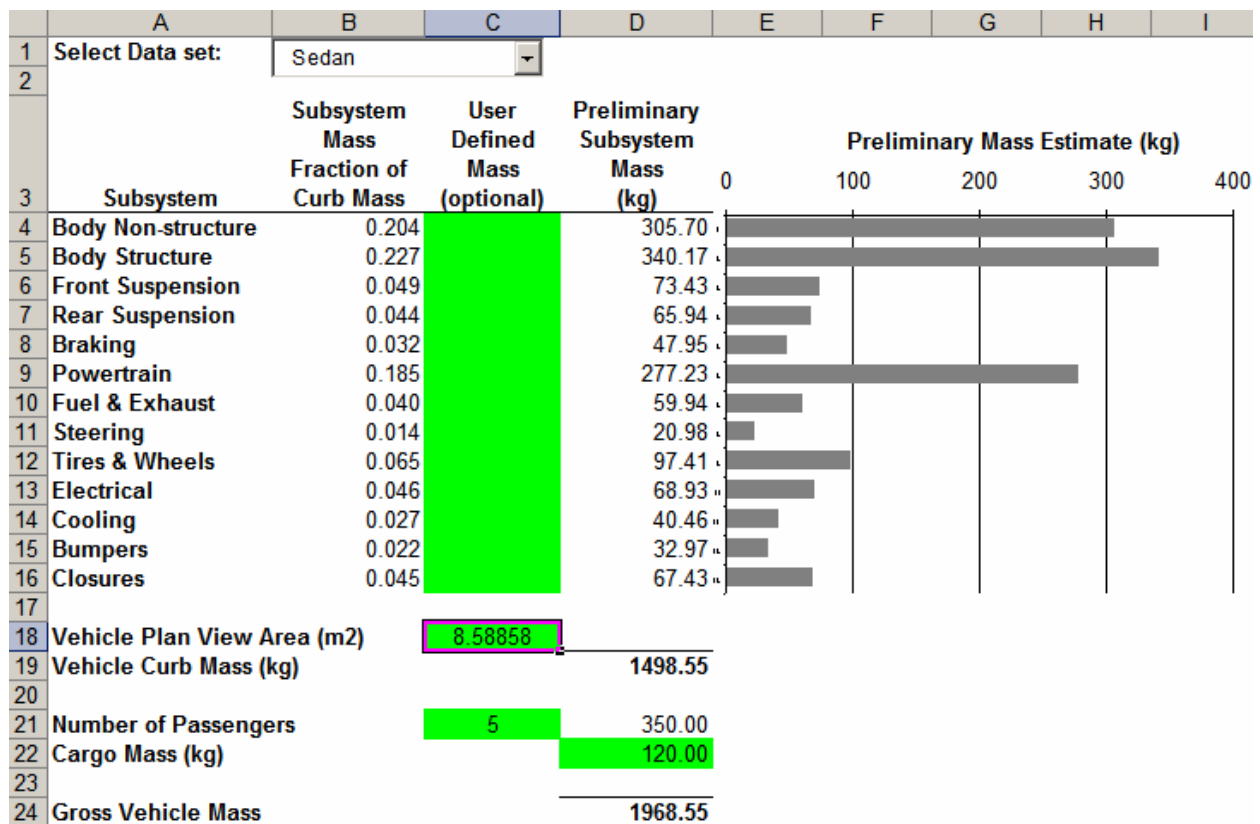
$$\text{curb mass} = 1498.55\text{kg}$$

Adding passenger mass and cargo mass, the gross vehicle mass is found. Finally, based on the *Subsystem Mass fraction of Curb Mass* (Appendix C in the Sedan Section) the mass of each subsystem may be estimated. For example,

$$(\text{Body Non-structure mass}) = 0.204 (\text{Curb Mass})$$

$$(\text{Body Non-structure mass}) = 305.70 \text{ kg}$$

These steps are automated in the spreadsheet provided with this project, Figure 8.1.



Pre-Configuration Vehicle Mass Estimation

Figure 8.1

The above mass estimate is for a nominal vehicle as defined by the data set used in this paper given the footprint. This estimate is greater than the desired 886kg curb mass by 633kg.

Step 2: Identify Mass Reduction Technologies

With the nominal vehicle identified, potential mass reduction technologies may be identified for each subsystem. For consistent evaluation of technologies, these steps are suggested;

2a) The initial mass for each subsystem is given by the nominal vehicle of Step 1 (those identified in Figure 8.1 in this example).

Example: Front Suspension subsystem mass=73.43kg

2b) Now for a vehicle of the gross vehicle mass identified in Step 1 (1968.55kg) determine the new subsystem mass enabled by a particular technology.

Example technology: Shape optimization arms

Front Suspension mass with technology=66.09kg

>>Note that this new front suspension is sized for the gross mass of the vehicle identified in Step 1. This is required for the later mass compounding analysis.<<

2c) The mass savings is the difference between 2a) and 2b).

Example mass savings=73.43-66.09=7.34kg

These steps are summarized for several hypothetical technologies in Table 8.1 Columns 1, 2, 3.

subsystem	mass reduction technology	mass savings	cost	cost/unit mass (mc)
		kg	\$	\$/kg
Tires & Wheels	Minimum capacity wheels and tires	20.00	-20	-1.000
non structure	reduced sound treatment with	19.45	-5	-0.257
Braking	optimized pedal bracket	3.00	0	0.000
rear suspension	shape optimization	6.59	0	0.000
body structure	joint improvements	15.00	0	0.000
closures	hardware: optimization for bar	10.00	0.1	0.010
front suspension	shape optimization	7.34	0.2	0.027
body structure	AHSS optimization	70.00	3	0.043
non structure	reduce glass thickness	5.00	0.3	0.060
non structure	IP substrate optimization	21.43	2	0.093
Braking	tubular pedals	4.00	0.4	0.100
non structure	seat frame shape optimization	40.00	5	0.125
closures	AHSS optimization	12.46	2	0.161
Fuel and Exhaust	lower gage of exhaust	5.99	1	0.167
bumper	shape optimization	4.95	1	0.202
steering	tubular rack	2.00	0.5	0.250
powertrain	reduce wall thickness	27.72	7	0.253
non structure	carpet material	15.00	5	0.333
non structure	new seat system technology	5.00	5	1.000
Fuel and Exhaust	New muffler technology	10.00	50	5.000
body structure	carbon fiber underbody	60.00	500	8.333

Potential Mass Reduction Technologies (Hypothetical Examples)

Table 8.1

It is important to note again that the mass reduction in Column 3 must be based on the vehicle mass identified in Step 1 to meet the assumptions of the mass compounding model.

Step 3: Sort Mass Reduction Technologies by Cost

This marginal cost of mass, mc , (cost per unit mass reduction) may be used to sort which technologies to include in the new vehicle program.

- 3a) Identify the cost difference to provide this technology in this specific vehicle. (Note, this may be a cost reduction and a negative value)

Example technology: Shape optimization arms

Front Suspension cost increase with technology=\$0.20

- 3b) Identify the marginal cost to provide the technology.

Example: marginal cost= $mc=(\$0.20)/(7.34\text{kg reduction})=0.027\$/\text{kg}$

- 3c) From all identified mass reduction technologies, sort by increasing marginal cost

Example: Table 8.1 is sorted for 21 potential technologies

- 3d) Filter technologies using marginal cost for inclusion in the vehicle configuration

Example: Include all technologies with $mc \leq 0.4 \text{ } \$/\text{kg}$

The 18 technologies at the top of Table 8.1 are carried forward.

3e) For those technologies passing the filter, sum the total mass reduction for each subsystem.

Example;

<u>Subsystem</u>	<u>Mass reduction using technologies passing mc filter</u>
Non Structure	19.45+5+21.43+40+15=100.88kg
Body Structure	15+70=85kg
Front Suspension	7.34kg
Rear Suspension	6.59kg
Braking	3+4=7.00kg
Powertrain	27.72kg
Fuel and exhaust	5.99kg
Steering	2.00kg
Tire & Wheels	20kg
Bumper	4.95kg
Closures	10.0+12.46=22.46kg

These technologies provide a primary mass reduction total of 289.93kg.

Step 4: Estimate Vehicle Mass Using Mass Compounding

The mass savings opportunities found in Step 3, are *primary* mass reductions. They are sized for the vehicle mass found in Step 1; 1500kg curb mass. Now as each technology is applied to the vehicle, the curb mass is reduced. Due to this reduction, the subsystems may be resized. This resizing results in a *secondary* mass savings given by Equation 4.2,

Primary mass savings $\Delta = 289.93\text{kg}$

Secondary mass savings = $\Delta \left[\frac{\gamma_v}{(1 - \gamma_v)} \right]$, where the vehicle influence coefficient, γ_v , may be based

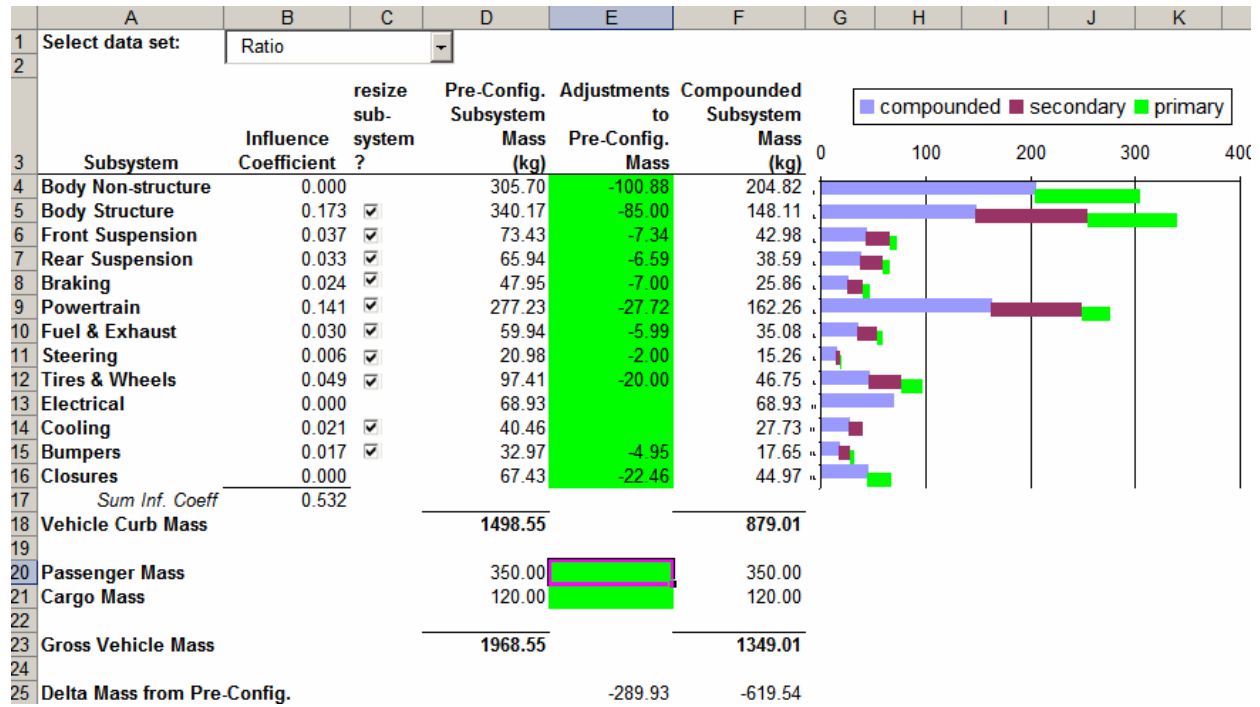
on the results of this study shown in Figure 7.1. Taking a nominal value for the vehicle influence coefficient; $\gamma_v = 0.532$ gives $\left[\frac{\gamma_v}{(1 - \gamma_v)} \right] = 1.1368$ or a secondary mass savings = 329.61kg.

Thus the total mass reduction when the mass reduction technologies are applied is the compounded vehicle mass which is the sum of the primary reduction (289.93kg) and the secondary reduction (329.61kg) due to the subsystems being resized for the new reduced vehicle mass; 289.93kg + 329.61kg = 619.54kg. To see how this compounded reduction is distributed across the subsystems, Equation 4.3 and 4.4 may be used. These steps are automated in the spreadsheet provided with this project, Figure 8.2.

The spreadsheet of Figure 8.2 applies the mass influence coefficients found in this work to arrive at the compounded masses. The user may select from the *All Vehicles*, *Sedan*, *SUV* groups, use the *Ratio* method, Equation 6.1, or input a *User Defined* set of coefficients. Selection of which to use is based on the subject vehicle type and the predicted gross vehicle mass. The predicted mass should be within the range of data used to estimate the influence coefficients. Interpolation limits for the influence coefficients are;

All Vehicles group	1600<GVM<3150kg
Sedan group	1600<GVM<2225kg
SUV group	1920<GVM<3150kg

If a prediction beyond these ranges is necessary, the *Ratio* method is recommended.

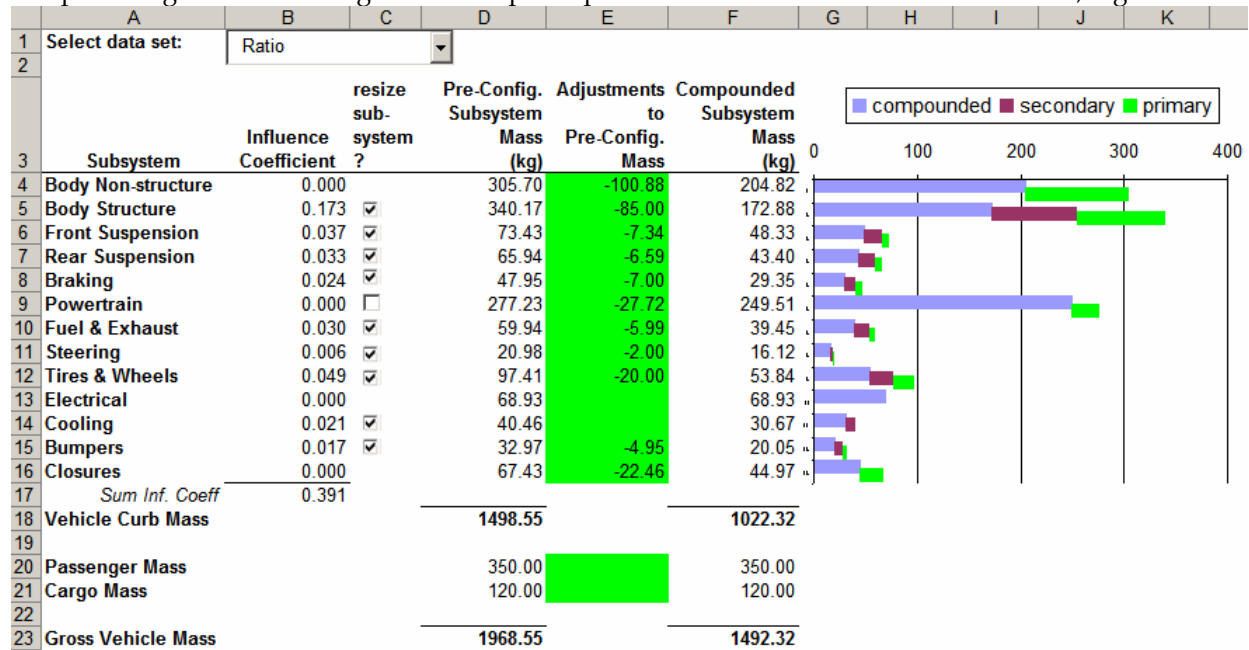


Compounded Mass Estimation (including powertrain resizing)

Figure 8.2

Step 5: Adjust Subsystem Mass to arrive at Design Targets

In Steps 1-4, we have used a rational procedure to estimate a vehicle's mass based on current practice, and then to estimate the influence of mass reduction technologies using a mass compounding model. It is important during this stage to investigate alternative configurations. For example, below is the compounding model as in Figure 8.2 except the powertrain has not been allowed to resize, Figure 8.2.



Compounded Mass Estimation (powertrain not resized)

Figure 8.2

These mass estimation tools are intended to be highly iterative allowing rapid sensitivity studies to arrive at robust design targets for vehicle and subsystem mass.

Bibliography

Past Influence Coefficient Studies

1. Internal GM communication, **Report of the Cost of Weight Task Force**, January, 1975
2. Internal GM communication, **Weight Influence Coefficients**, November, 1981.
3. Anderson, M., *Engine and Transmission Mass as a Fraction of Vehicle Mass*, **Integrated Vehicle System Design Course Notes**, University of Michigan, January, 2007.

Independent Mass Data Sources

4. www.autobytel.com, Autobytel, 18872 MacArthur Boulevard, Irvine, California 92612-1400.
5. www.cars.com, Classified Ventures, LLC, 175 W. Jackson Blvd., Suite 800, Chicago, IL 60604.
6. www.internetautoguide.com, Internet Auto Guide.

Methodology for Mass Analysis

7. Society of Allied Weight Engineers, **Aircraft Weight Engineering**, 1996
8. Tornbeek, E., **Synthesis of Subsonic Airplane Design**, Delft University Press, Boston, 1988
9. de Weck, O. L., *A Systems Approach To Mass Budget Management*, Paper AIAA 2006-7055, **11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference**, September 2006.
10. **Spacecraft Preliminary Mass Estimation & Allocation**, Technical University of Delft, <http://www.lr.tudelft.nl/live/pagina.jsp?id=10d9b8b2-e85b-48e7-94f2-462356f1d730&lang=en>
11. **Piano Software For Preliminary Design Of Commercial Aircraft Including Mass Estimation**, <http://www.lissys.demon.co.uk/>

Statistical Methods

12. Box, G., and Draper, N., **Empirical Model-Building and Response Surfaces**, Wiley & Sons, NY, 1987
13. Mendenhall, W., et al, **Mathematical Statistics with Applications**, Duxbury Press, Boston, 1986.
14. Neter, J., et al., **Applied Linear Statistical Models**, IRWIN Publishing, Boston, 1990

Technical Appendix A

Derivation of mass model and influence coefficients

Vehicle mass is the sum of the masses for n subsystems,

$$W_V = w_1 + w_2 + \dots + w_i + \dots + w_n \quad \text{Equation A.1}$$

Where W_V is the vehicle mass

w_i is the mass of subsystem i

For convenience, we include as subsystems the passengers, and cargo, so W_V represents the maximum vehicle mass (gross vehicle mass).

The mass of each subsystem depends upon the mass of the vehicle and other functional parameters.

$$w_i = f(W_V, \text{functional parameters}) \quad \text{Equation A.2}$$

Note that some subsystems will have a strong vehicle mass dependence—for example, the powertrain subsystem—while for others the dependence on vehicle mass will be weak—for example, interior ventilation.

Mass Influences Coefficients

Imagine subsystem i is designed to function within a vehicle mass of W_V with the resulting subsystem mass of w_i . Now imagine a second vehicle which is slightly heavier at mass $W_V + dW_V$. Subsystem i is now sized for this slightly larger mass with the resulting subsystem mass of $w_i + dw_i$. We define the subsystem mass influence, γ_i , as the change in subsystem mass required when the vehicle mass is increased by one unit;

$$\gamma_i = \frac{\partial w_i}{\partial W_V} \quad \text{Equation A.3}$$

Consider a vehicle design with initial mass W_0 and all subsystems sized for this mass. Now during preliminary design, an unexpected small mass increase of δ occurs in a subsystem. All subsystems must now be resized for this mass increase. The resulting change in vehicle mass is dW which we would like to determine. This can be quantified using subsystem mass influence coefficients;

$$\begin{aligned} dW &= \frac{\partial w_1}{\partial W_V} \delta + \frac{\partial w_2}{\partial W_V} \delta + \dots + \frac{\partial w_i}{\partial W_V} \delta + \dots + \frac{\partial w_n}{\partial W_V} \delta \\ dW &= \delta(\gamma_1 + \gamma_2 + \dots + \gamma_i + \dots + \gamma_n) \\ \frac{dW}{\delta} &= (\gamma_1 + \gamma_2 + \dots + \gamma_i + \dots + \gamma_n) \end{aligned} \quad \text{Equations A.4}$$

The vehicle mass influence coefficient, γ_V , is defined as the change in mass of the vehicle, dW , due to a resizing of all the subsystems for a unit increase in mass.

$$\gamma_V = \frac{dW}{\delta} \quad \text{Equation A.5}$$

Comparing the last of Equations A.4 with A.5, we see that the vehicle mass influence coefficient, γ_v , is the sum of the influence coefficients for all subsystems.

$$\gamma_v = \sum_{i=1}^n \gamma_i \quad \text{Equation A.6}$$

Mass Change during the Configuration Design Stage

Now consider a balanced vehicle design with initial mass W_0 . It is now learned that the mass of a subsystem has changed by an amount Δ for a new vehicle mass of $W_0 + \Delta$.

Since the subsystems were sized for vehicle mass W_0 , they must be redesigned to function with the additional mass Δ :

Resizing 1

mass change : Δ

resize subsystems due to mass change causes additional mass : $\Delta\gamma_v$

new vehicle mass $W_1 = (W_0 + \Delta) + \Delta\gamma_v$

But now the subsystems are sized for vehicle mass $(W_0 + \Delta)$, and they must be resized for the additional mass $\Delta\gamma_v$:

Resizing 2

mass change : $\Delta\gamma_v$

resize subsystems due to mass change causes additional mass : $\Delta\gamma_v^2$

$W_2 = (W_0 + \Delta + \Delta\gamma_v) + \Delta\gamma_v^2$

In a similar way, the subsystems are sized for vehicle mass $(W_0 + \Delta + \Delta\gamma_v)$, and they must be resized for the additional mass $\Delta\gamma_v^2$:

Resizing 3

mass change : $\Delta\gamma_v^2$

resize subsystems due to mass change causes additional mass : $\Delta\gamma_v^3$

$W_3 = (W_0 + \Delta + \Delta\gamma_v + \Delta\gamma_v^2) + \Delta\gamma_v^3$

This resizing continues until we have:

Resizing m

$$W_m = W_0 + \Delta + \Delta\gamma_v + \Delta\gamma_v^2 + \Delta\gamma_v^3 + \dots + \Delta\gamma_v^j + \dots + \Delta\gamma_v^m, \quad m \rightarrow \infty$$

Equations A.7

$$W_m = W_0 + \Delta + \Delta(\gamma_v + \gamma_v^2 + \gamma_v^3 + \dots + \gamma_v^j + \dots + \gamma_v^m), \quad m \rightarrow \infty$$

Recognizing the sum in the parentheses is a geometric series we have;

$$\begin{aligned}
S &= (\gamma_v + \gamma_v^2 + \gamma_v^3 + \dots + \gamma_v^j + \dots + \gamma_v^m), \quad m \rightarrow \infty \\
S &= \gamma_v (1 + \gamma_v + \gamma_v^2 + \gamma_v^3 + \dots + \gamma_v^j + \dots + \gamma_v^m), \quad m \rightarrow \infty \\
S &= \gamma_v (1 + S) \\
S(1 - \gamma_v) &= \gamma_v \\
S &= \frac{\gamma_v}{(1 - \gamma_v)}, \text{ when } 0 < \gamma_v < 1
\end{aligned}
\tag{Equations A.8}$$

Substituting the last of Equations A.8 into A.7 gives the mass of the vehicle after all resizing;

$$W_{m \rightarrow \infty} = W_0 + \Delta + \Delta \left[\frac{\gamma_v}{(1 - \gamma_v)} \right] \tag{Equations A.9}$$

Where Δ is the original mass change

$$\Delta \left[\frac{\gamma_v}{(1 - \gamma_v)} \right] \text{ is the secondary mass change due to resizing subsystems.}$$

We define the *Secondary mass coefficient* for the vehicle, Γ_v ;

$$\Gamma_v = \left[\frac{\gamma_v}{(1 - \gamma_v)} \right] \tag{Equation A.10}$$

and for each subsystem, Γ_i , as;

$$\Gamma_i = \left[\frac{\gamma_i}{(1 - \gamma_i)} \right] \tag{Equation A.11}$$

So the final vehicle mass due to an unexpected mass increase of Δ is

$$W_{v\infty} = W_0 + \Delta + \Delta \Gamma_v \tag{Equation A.12}$$

and the final mass for subsystem i is;

$$w_{i\infty} = w_i + \Delta \Gamma_i \tag{Equation A.13}$$

To prove that the sum of subsystem influence coefficients is the vehicle influence coefficient, begin with Equation A.12;

$$\begin{aligned}
W_{m \rightarrow \infty} &= W_0 + \Delta + \Delta \Gamma_v \\
W_v &= w_1 + w_2 + \dots + w_i + \dots + w_n \\
W_{m \rightarrow \infty} &= \Delta + (w_1 + \Delta \Gamma_1) + (w_2 + \Delta \Gamma_2) + \dots + (w_i + \Delta \Gamma_i) + \dots (w_n + \Delta \Gamma_n) \\
W_{m \rightarrow \infty} &= \Delta + (w_1 + w_2 + \dots + w_i + \dots + w_n) + \Delta(\Gamma_1 + \Gamma_2 + \dots + \Gamma_i + \dots + \Gamma_n) \\
W_{m \rightarrow \infty} &= \Delta + W_0 + \Delta(\Gamma_1 + \Gamma_2 + \dots + \Gamma_i + \dots + \Gamma_n) \\
\Delta \Gamma_v &= \Delta(\Gamma_1 + \Gamma_2 + \dots + \Gamma_i + \dots + \Gamma_n) \\
\Delta \left[\frac{\gamma_v}{(1 - \gamma_v)} \right] &= \Delta \left[\frac{\gamma_1}{(1 - \gamma_v)} + \frac{\gamma_2}{(1 - \gamma_v)} + \dots + \frac{\gamma_i}{(1 - \gamma_v)} + \dots + \frac{\gamma_n}{(1 - \gamma_v)} \right] \\
\gamma_v &= \gamma_1 + \gamma_2 + \dots + \gamma_i + \dots + \gamma_n
\end{aligned}
\tag{Equation A.14}$$

Technical Appendix B

Modeling Subsystem Mass

We assume that the mass of each subsystem depends on the gross vehicle mass and on other functional parameters [8];

$$w_i = f(W_V, \text{functional parameters})$$

$$w_i = C_i W_V^\alpha (P_a^{\beta_a} P_b^{\beta_b} \dots P_r^{\beta_r})$$
Equation B.1

Where w_i is a subsystem mass

W_V is the gross vehicle mass

$P_{a,b,\dots,r}$ are functional parameters some of which are performance measures

For example, consider the powertrain subsystem mass, w_i . Its mass depends on the overall vehicle mass, $C_i W_V^\alpha$. The mass also depends on vehicle acceleration performance; $P_a^{\beta_a}$, where P_a is the required 0-60mph acceleration time. It also depends on the powertrain layout; TFWD, LRWD, AWD; P_b . Here P_b is an indicator variable: $P_b = +1$ for TFWD, $P_b = 0$ for LRWD, $P_b = -1$ for AWD.

This relationship may be visualized graphically;

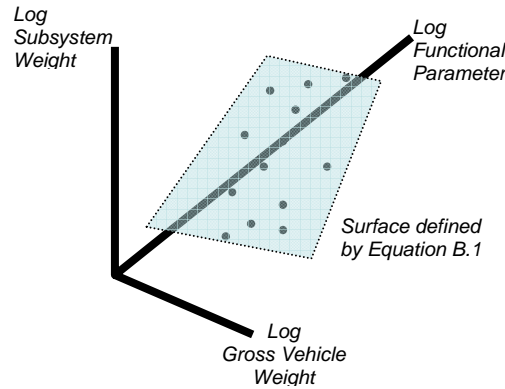


Figure B.1

With the model defined by Equation B.1, the subsystem influence coefficient is given by;

$$\gamma_i = \left. \frac{\partial w_i}{\partial W_V} \right|_{W_V}$$
Equation B.2

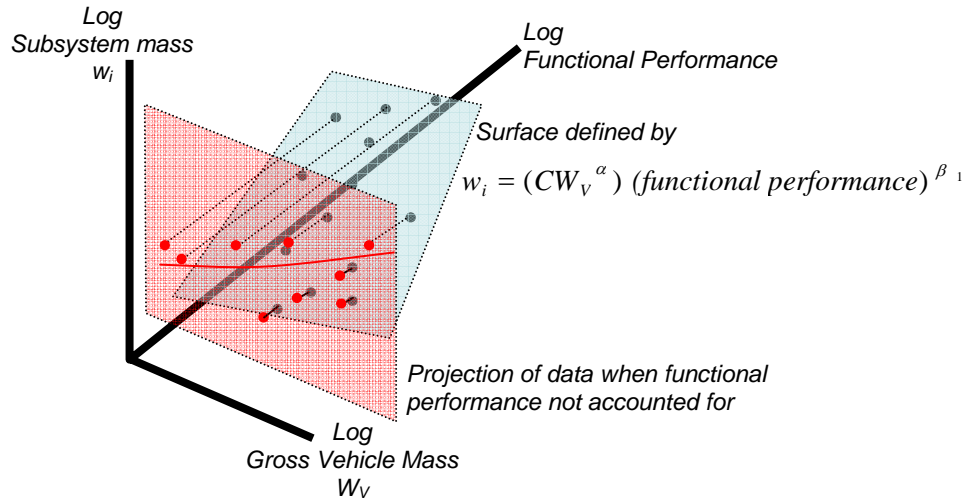
Estimation of subsystem influence coefficients

To estimate the relationship between subsystem mass and vehicle mass, we fit observed data with both a linear and power model. In both cases, the fit was made using linear regression.

$$\hat{w}_i = C_i W_V^\alpha \quad \text{power}$$

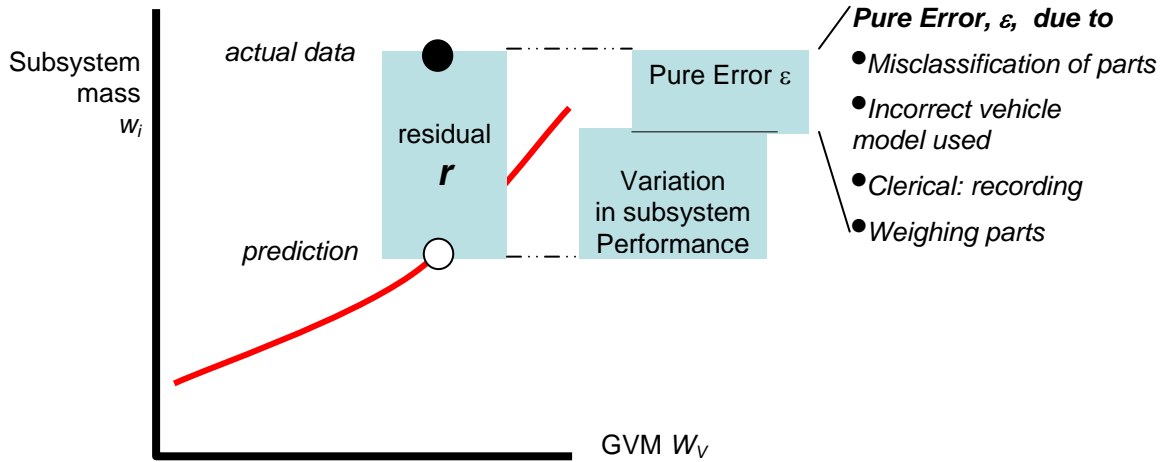
$$\hat{w}_i = \beta_0 + \beta_1 W_V \quad \text{linear}$$
Equations B.3

In this study, we have not accounted for the subsystem mass dependence on functional parameters and have only considered the dependence on gross vehicle mass. Thus we are looking at a projection of the true relationship onto an x-y plane, Figure B.2.



Projection of data on to x-y plane for curve fitting
Figure B.2

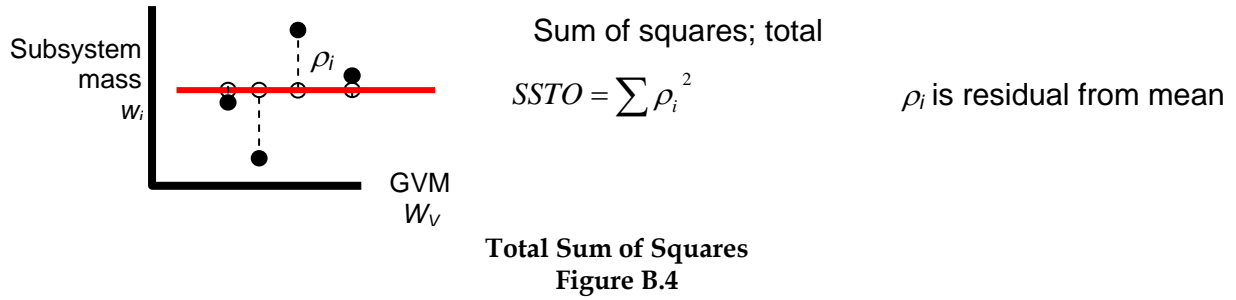
This necessity, has the practical effect of increasing the apparent lack of fit (measured by smaller R^2 values). Figure B.3 illustrates this by showing a typical residual error—the difference between a data point and the fit function. In our case, this residual is comprised of both pure error—randomness, along with the ignored dependence on functional parameters. Despite this assumption, the data fit is deemed to be satisfactory even though the R^2 values are somewhat low due to this effect.



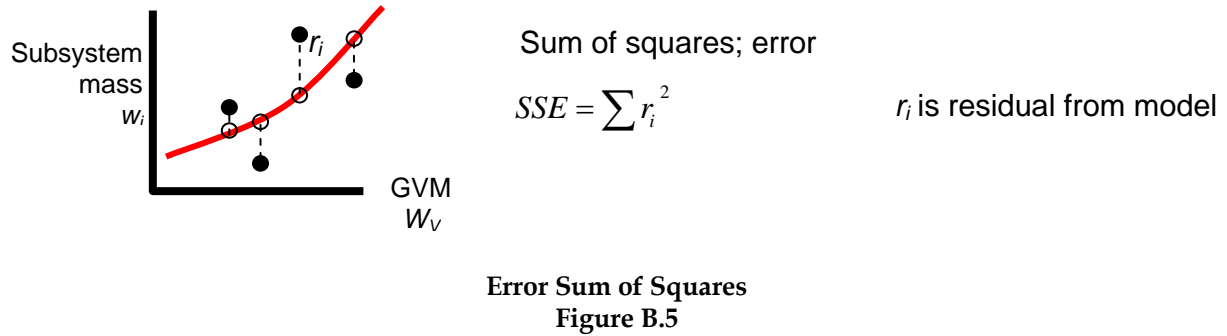
Components of Residual Error
Figure B.3

For all fit equations, the R^2 values are reported. Below is a brief summary of the meaning of this coefficient [12, 14].

The variation of the un-fit data is measured by the sum of the squared deviations from the *average* value, Figure B.4. This sum is the total sum of squares, SSTO.



Now a function is fit to the data as illustrated in Figure B.5. The remaining variation in the data is measured by the sum of the squared deviations from the *function*. This sum is the error sum of squares, SSE.



The R^2 coefficient of determination is the fraction of variation explained beyond that using the average value only, and is given by;

$$R^2 = \frac{(SSTO - SSE)}{SSTO} \quad \text{Equation B.4}$$

Thus any non-zero R^2 indicates a reduction in variation provided by the fit function. Again in our case, the residual error, r_i , contained in the SSE contains both pure error as well as the variation due to functional performance differences between the various vehicles as shown in Figure B.3.

While R^2 measures the overall goodness of fit of the model, we are interested in the influence coefficient. For a linear model, the influence coefficient is the slope of the fit line, β_1 . We can place a confidence interval around this parameter using Equation B.5 [13];

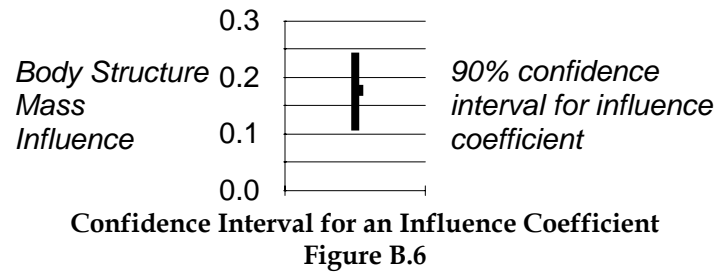
$$\begin{aligned} \text{data} &: x_1, x_2, x_3, \dots, x_n; y_1, y_2, y_3, \dots, y_n; \\ \hat{y} &= \beta_0 + \beta_1 x \\ \beta_1 &\sim N[\bar{\beta}_1, \text{Var}(\beta_1)] \\ \text{Var}(\beta_1) &= \left(\frac{1}{n-2} \right) \frac{\sum (\hat{y} - y)^2}{\sum_{i=1,n} (x - \bar{x})^2} \end{aligned} \quad \text{Equations B.5}$$

For example, looking at the *All Vehicles* group and the *Body Structure* subsystem, we have an influence coefficient $\beta_1=0.1758$ and $R^2=0.4542$. Applying Equations B.5,

$$\begin{aligned}\hat{y} &= \beta_0 + \beta_1 x \\ \hat{y} &= -1.5294 + 0.1758x \\ n &= 33 \\ \text{Var}(\beta_1) &= .001198\end{aligned}$$

Equation B.6

The 90% confidence interval is shown in Figure B.6.



Data Appendix C

Vehicles used in analysis

Mass data were collected for 35 vehicles representing Sedans, SUVs, Pickups and Vans. For each vehicle, the mass of each functional subsystem (see next page for subsystem definitions) was calculated. Considerable effort was placed on consistency of the component masses contained in each subsystem across all vehicle sources. In addition to the subsystem mass, *Gross Vehicle Mass*, *Curb Mass*, and *Number of Passengers* were identified for each vehicle using independent sources [4, 5, 6].

The vehicles in this study are shown below. Due to the propriety nature of subsystem mass data, specific numerical values are not provided.

Sedans: 15 (of which there is 1 duplicate)
 SUV: 12
 Pick Up: 5

Note: Due to the small sample size, Pick Up results are less reliable than for Sedans and SUVs.

Van: 3 (of which there is 1 duplicate)
 35 (33 vehicles used in study)

Note: Due to the insufficient sample size, vans were not included as a vehicle group.

Note: For the 'All Vehicles' group, all non-duplicate vehicles were included

2004 VW Touareg	(SUV)	2002 Honda Civic LX	(Sedan)
2004 Mazda 3	(Sedan)	2003 Honda Accord EX	(Sedan)
2004 Nissan Murado	(SUV)	2003 PT Cruiser	(SUV)
2004 Toyota Sienna	(Van) (Dup.)	2003 Toyota Matrix XRS	(SUV)
2004 Hyundai XG350	(Sedan)	2003 Toyota Tacoma 4x2	(Pick Up)
2004 Toyota Prius	(Sedan)	2004 Dodge Ram 4x4 Light Duty	(Pick Up)
2003 Lexus ES300	(Sedan)	2004 Nissan Titan LE	(Pick Up)
2003 Toyota Camry (US)	(Sedan)	2004 Toyota Highlander Premium	(SUV)
2003 BMW 330i	(Sedan)	2004 Toyota Sienna	(Van)
2003 Infiniti G35	(Sedan)	2005 Honda Odyssey Touring	(Van)
2003 Honda Accord	(Sedan)(Dup.)	2005 Jeep Liberty	(SUV)
2003 Toyota Corolla Sedan	(Sedan)	2005 Jeep Wrangler	(SUV)
2002 Audi A4	(Sedan)		

2007 Model Year

Cadillac SRX	(SUV)
Chevrolet HHR	(SUV)
Saturn Outlook	(SUV)
GMC Denali Sierra Crew Cab	(Pick Up)
Chevrolet Colorado	(Pick Up)
Chevrolet Impala	(Sedan)
Pontiac G6 SE1	(Sedan)
Cadillac STS	(Sedan)
GMC Yukon	(SUV)
Saturn Vue	(SUV)

Functional Subsystem Mass Categories

Each category below contains the set of components which provide a specific vehicle function. The assumption is that the mass required to provide that function varies in part with the gross vehicle mass.

1. Body Non-structural

- Sheet Metal
- Glass
- Seats
- Insulation
- Trim
- Heating and Ventilation
- Exterior Lighting
- Wiper

2. Body Structure

- Body Shell (Body-in-White less closures)
- Frame (if present)
- Engine Cradle (if present)

3. Front Suspension

- Spring
- Control Arms
- Knuckle
- Stabilizer Bar

4. Rear Suspension

5. Braking Disc/Drum
 Caliper
 Hydraulic cylinder

6a. Engine Engine
 Engine Cooling
 Starting System

6b. Transmission

- Transmission
- Drive Shafts

Note: Engine and Transmission subsystems were combined to form the Powertrain Subsystem used in the final analysis.

7. Fuel System and Exhaust

8. Steering Rack
 Column
 Tie rods
 Power assist

9. Tires & Wheels

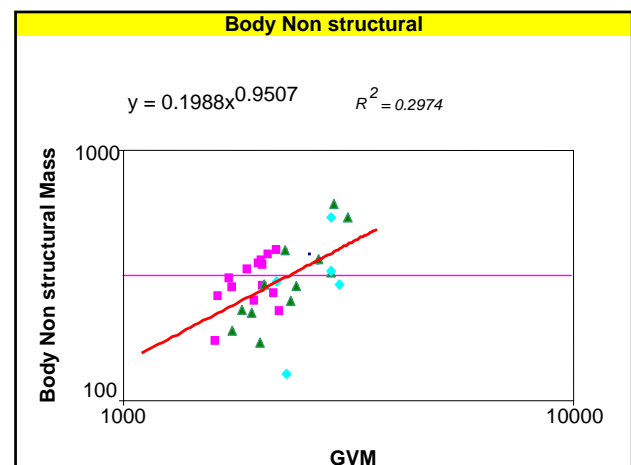
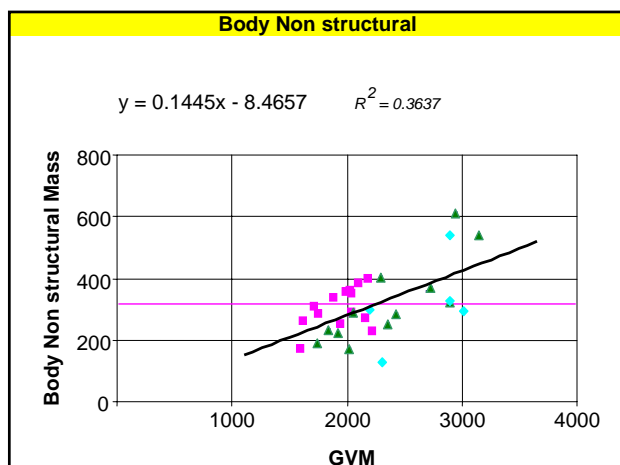
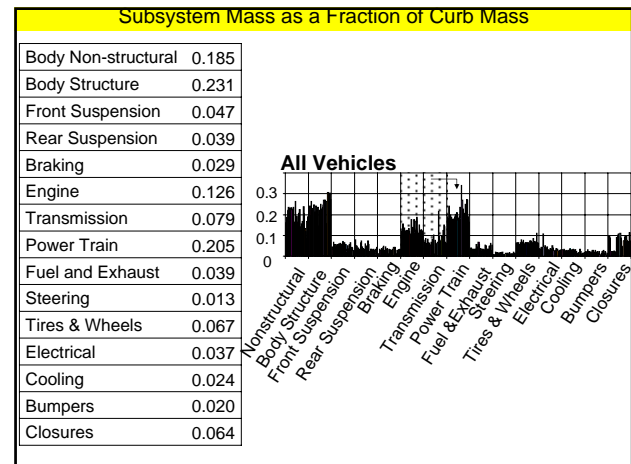
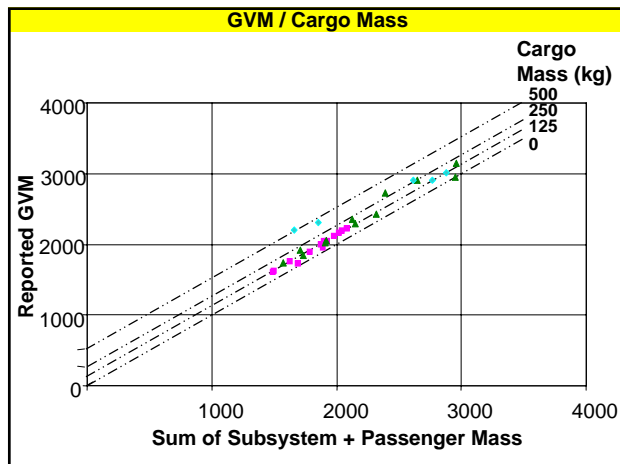
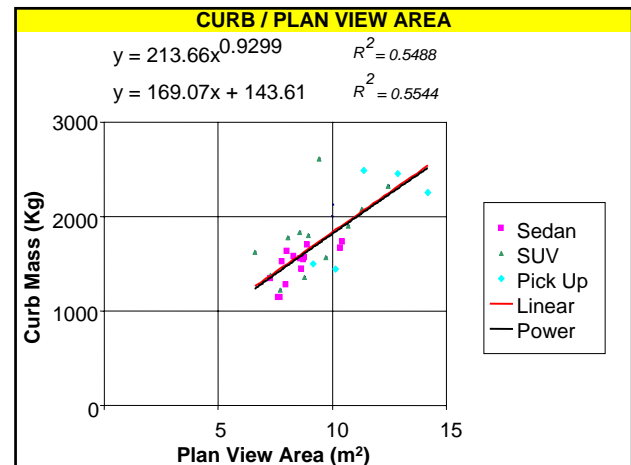
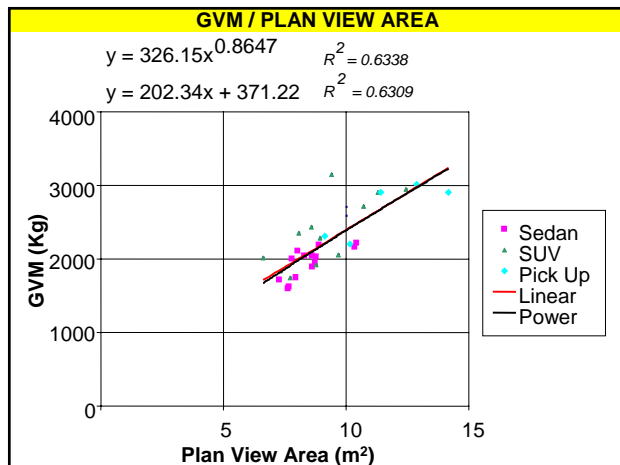
10. Electrical Entertainment, navigation
 Lighting
 Wiring

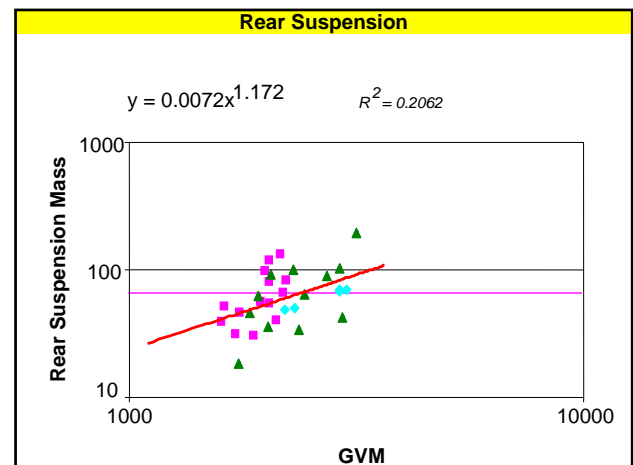
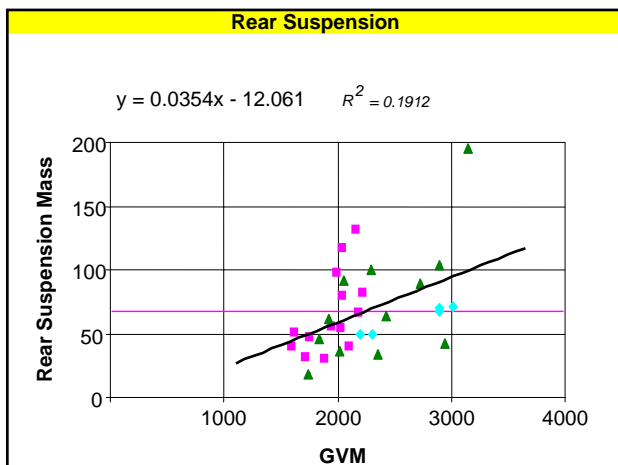
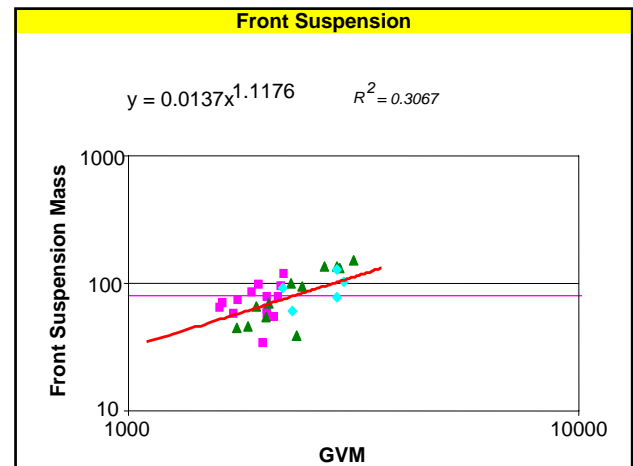
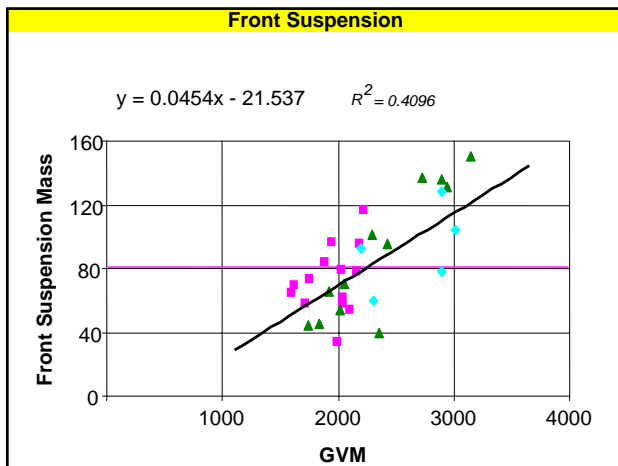
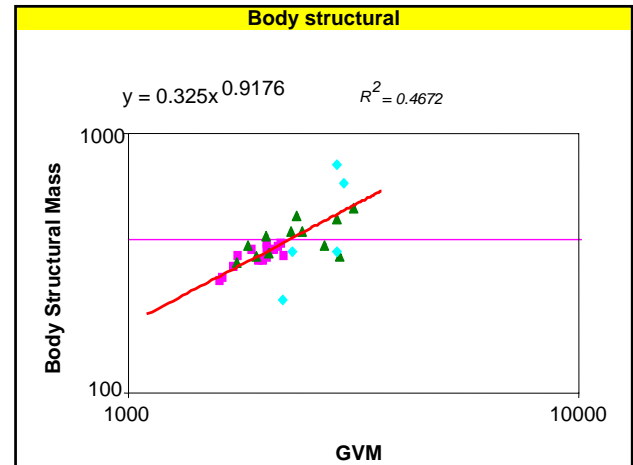
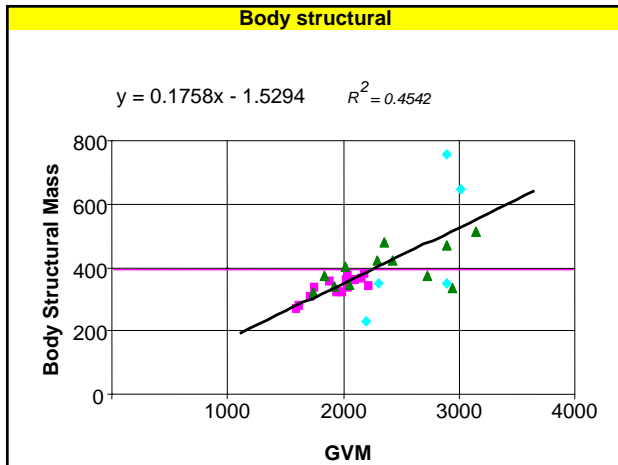
11. Cooling Air Conditioning components

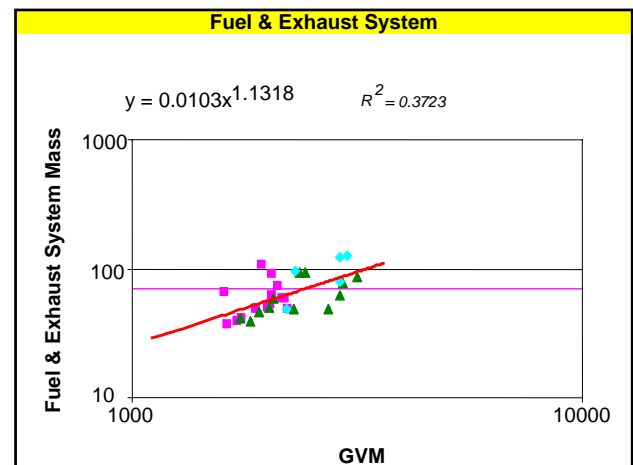
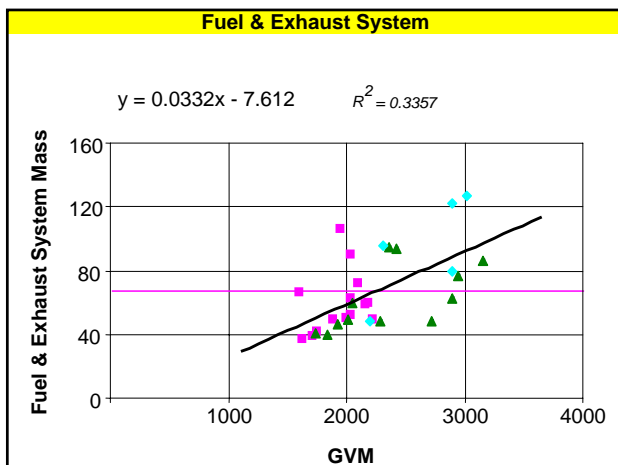
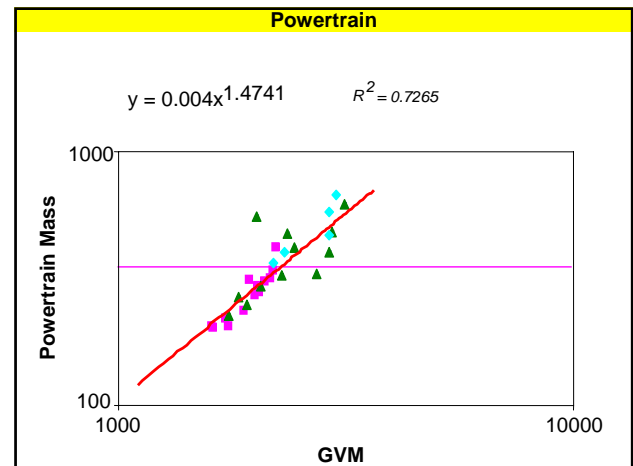
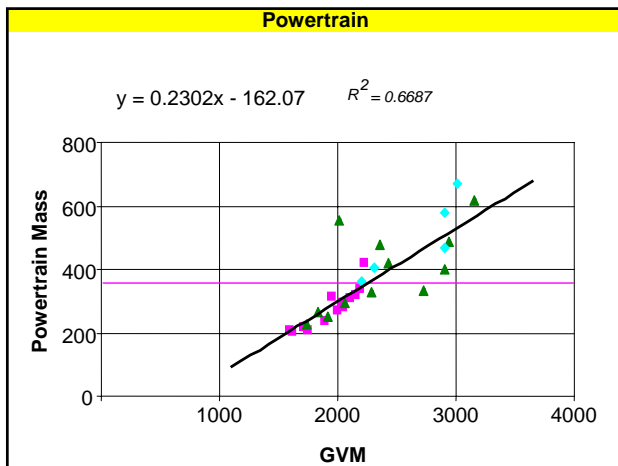
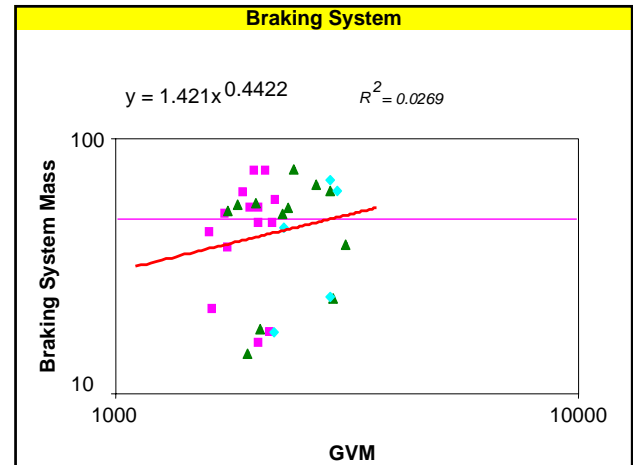
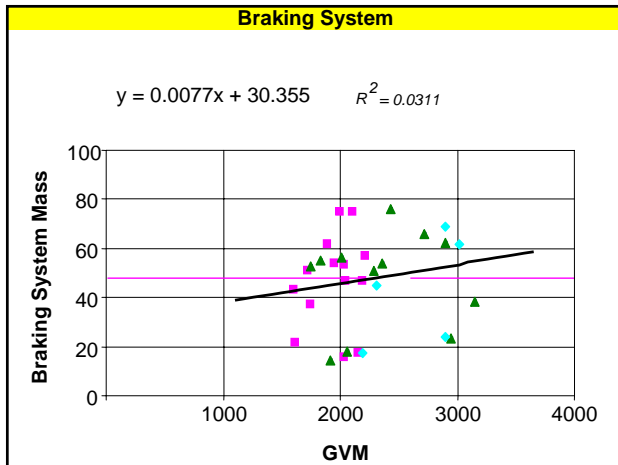
12. Bumpers

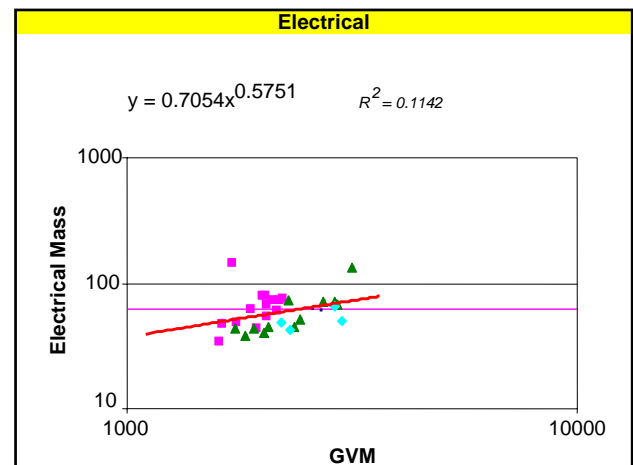
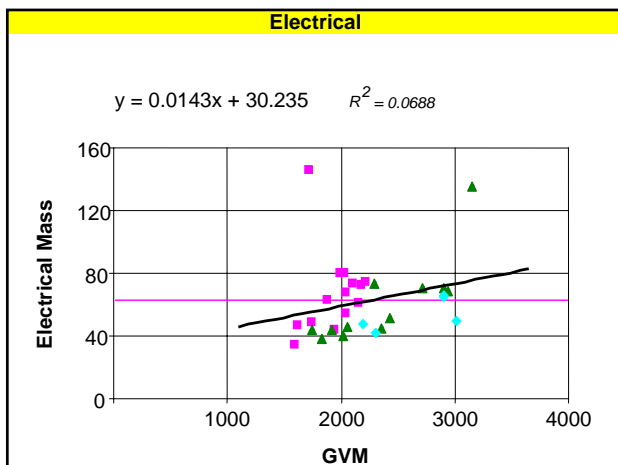
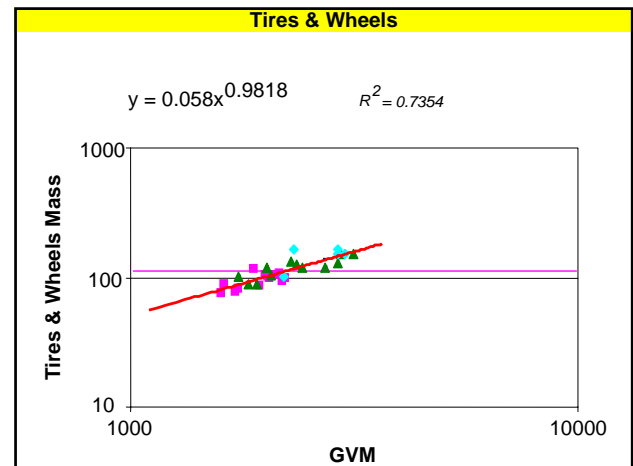
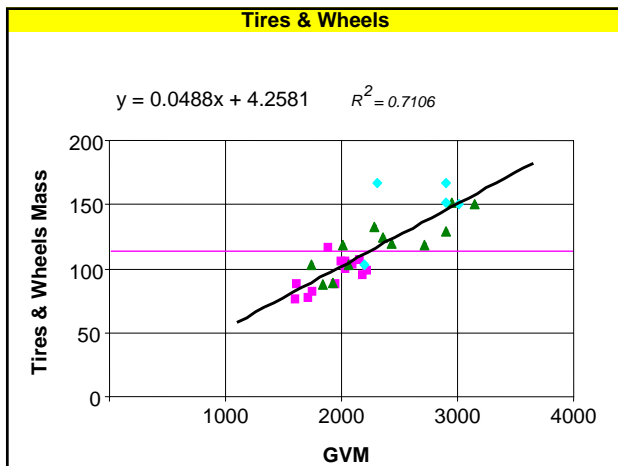
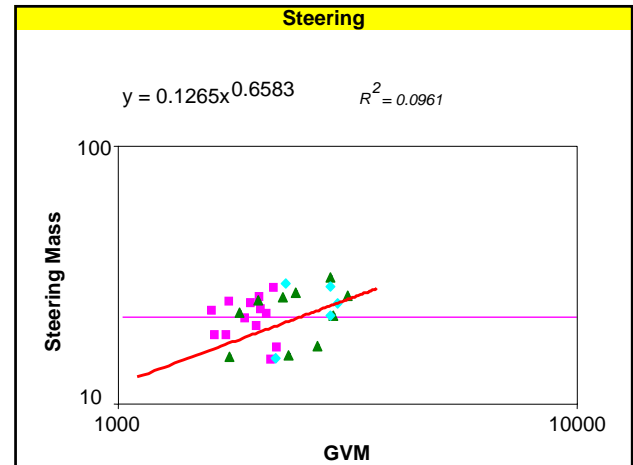
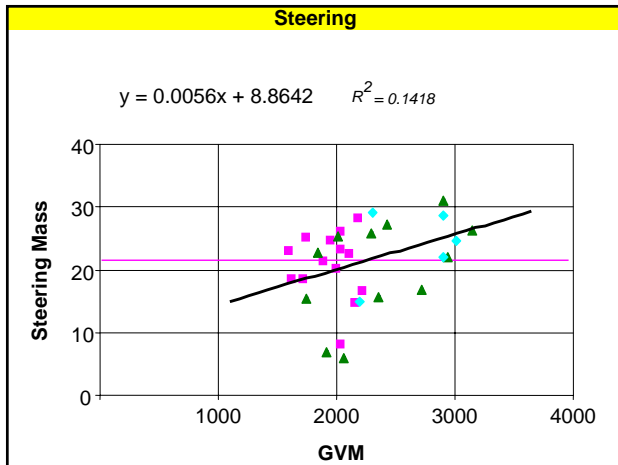
13. Body Closures

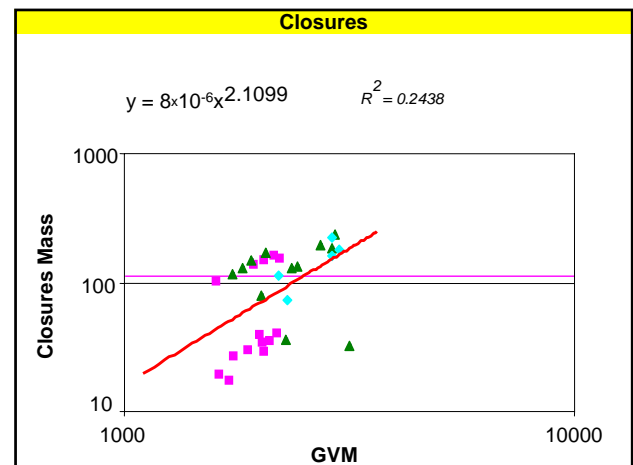
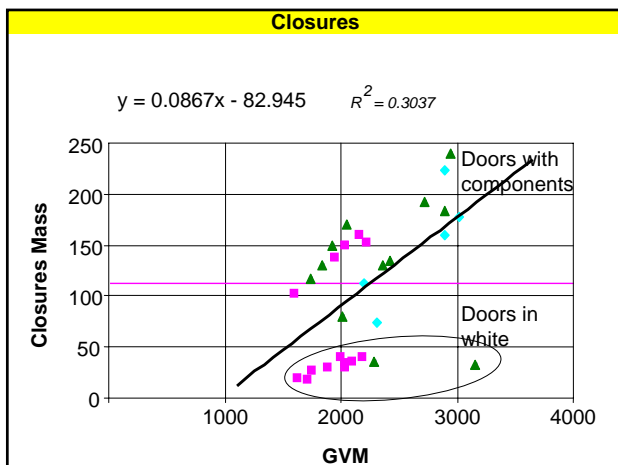
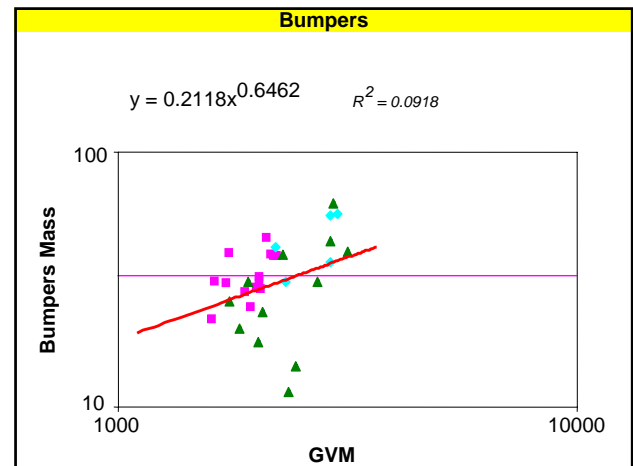
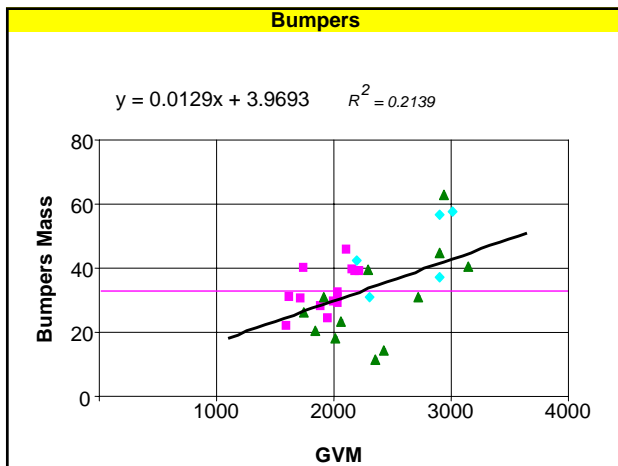
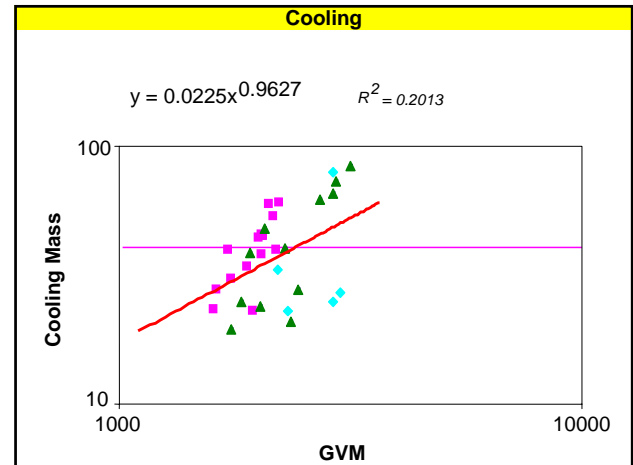
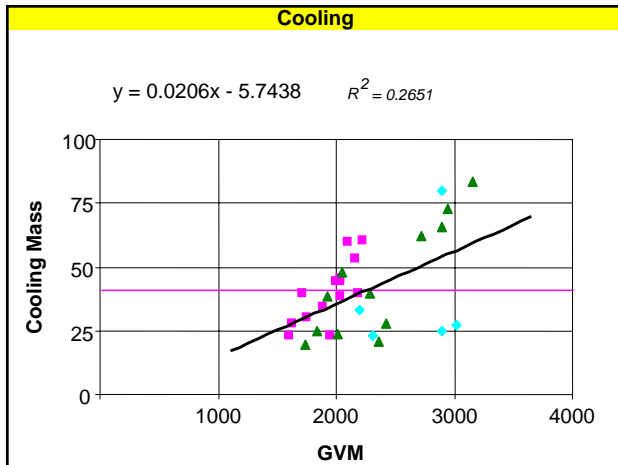
Note: Closures include hardware and door trim, with the exception of the GM Benchmark data where it is the door shell only. These cases are noted on the graphs.

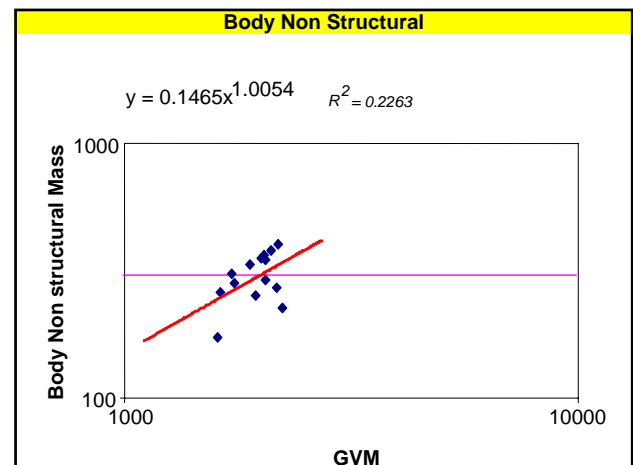
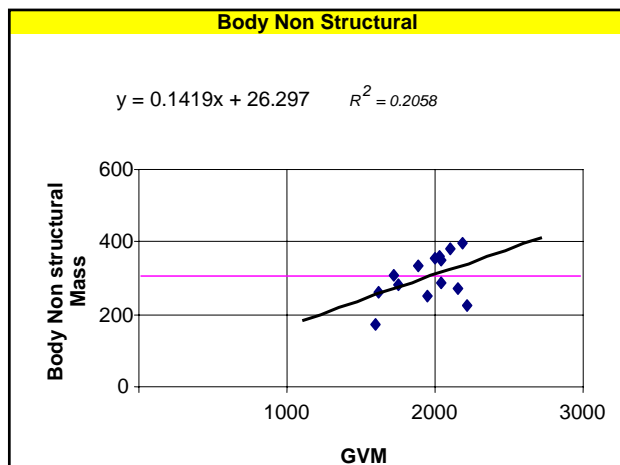
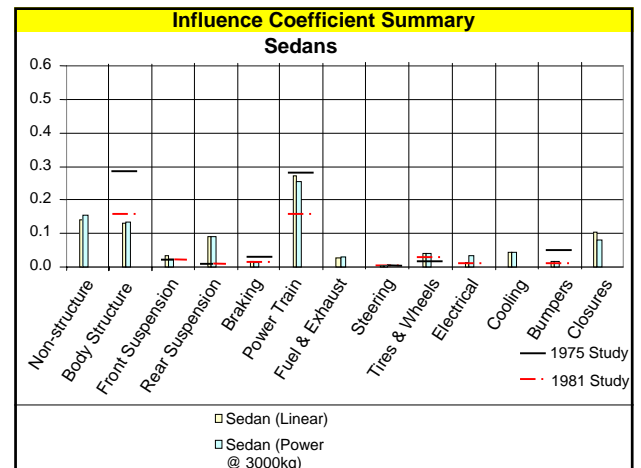
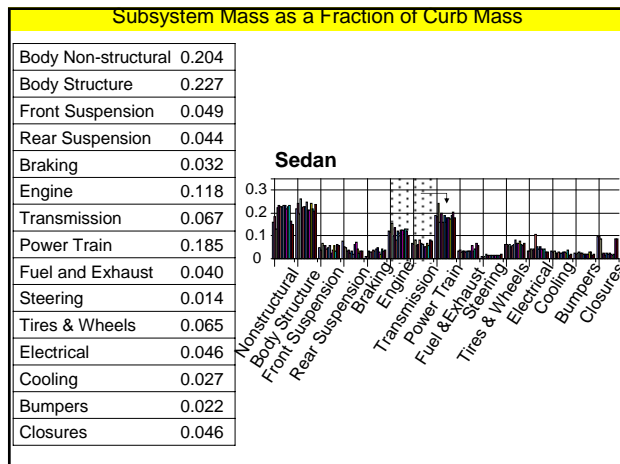
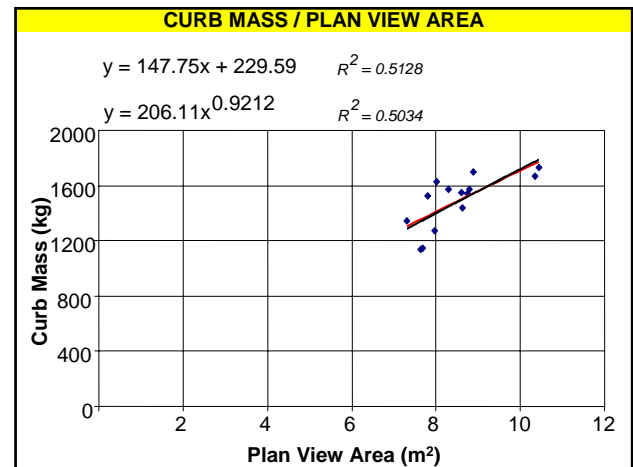
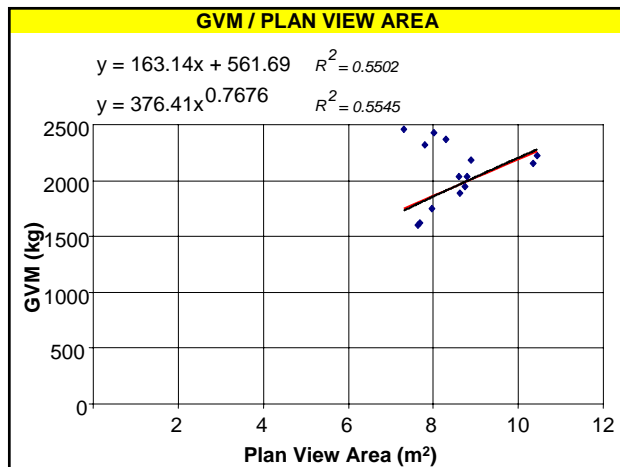


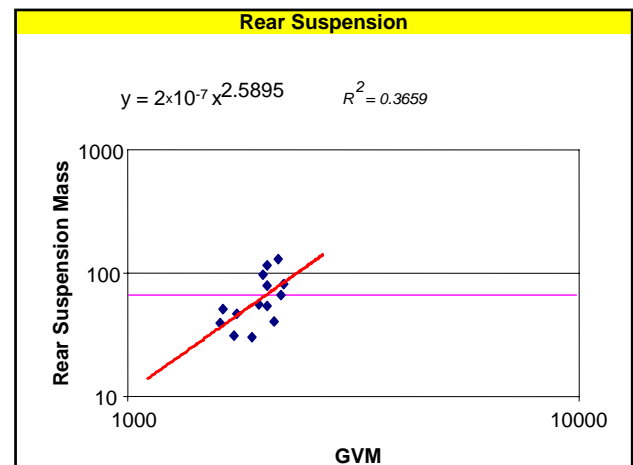
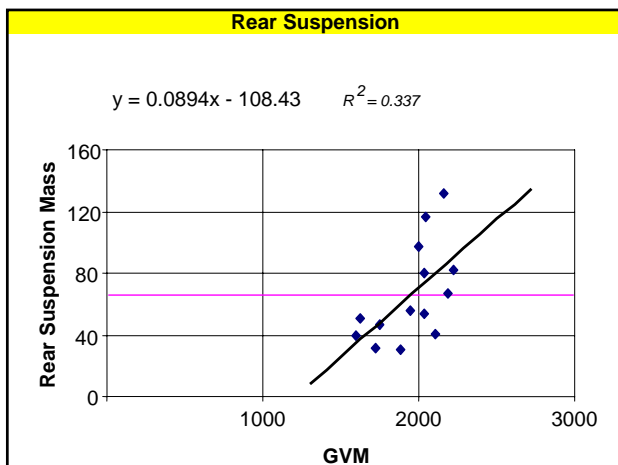
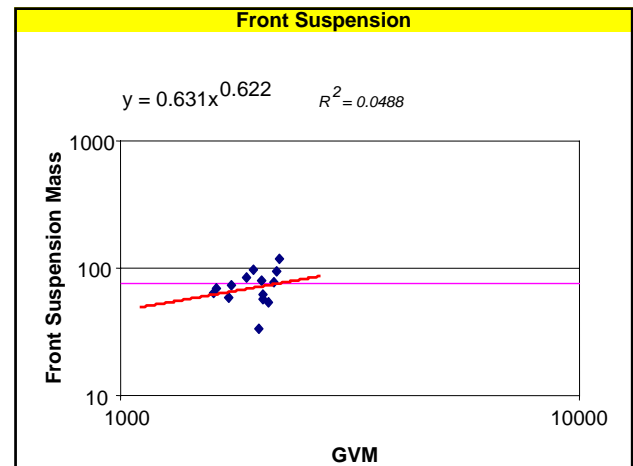
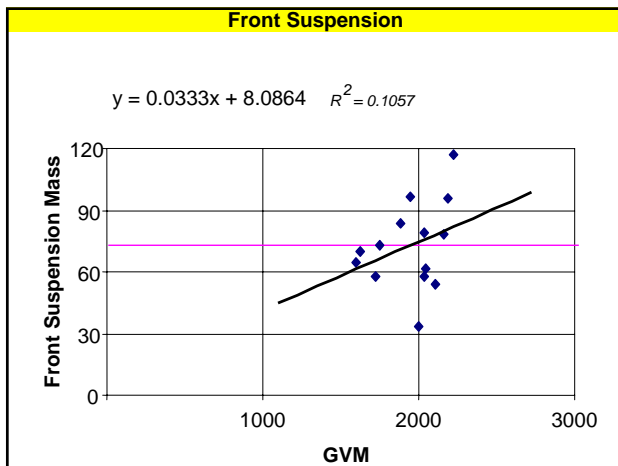
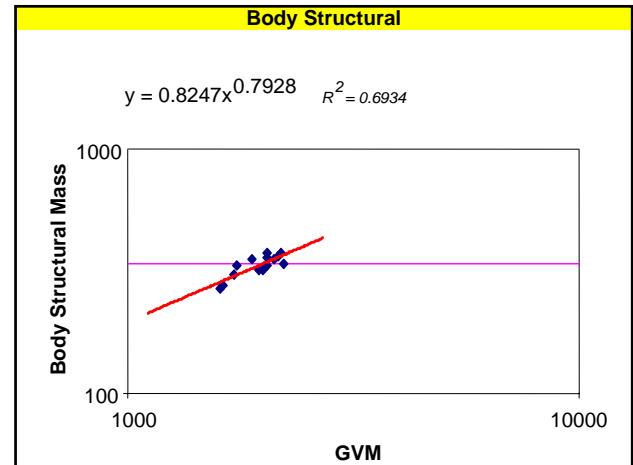
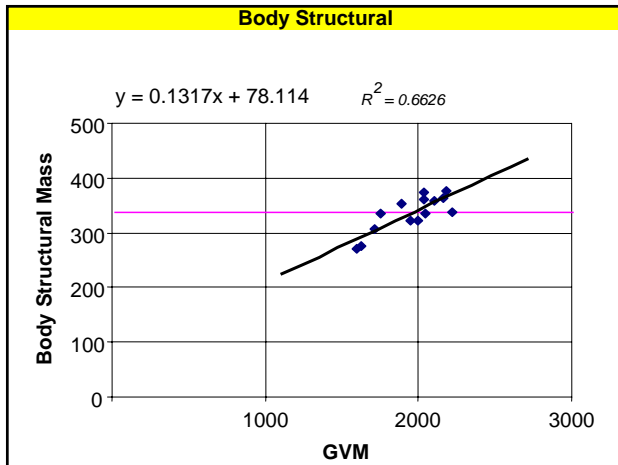


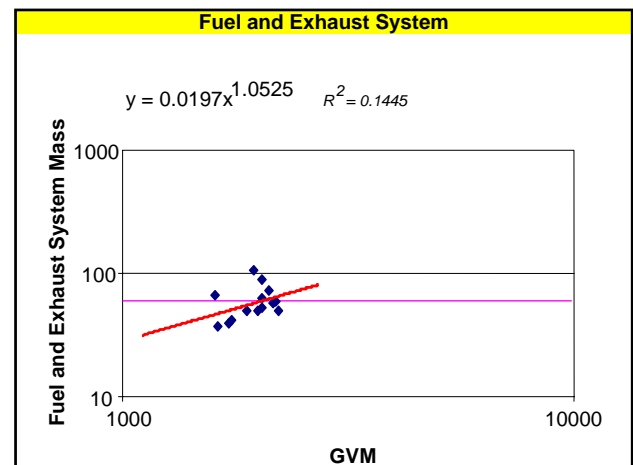
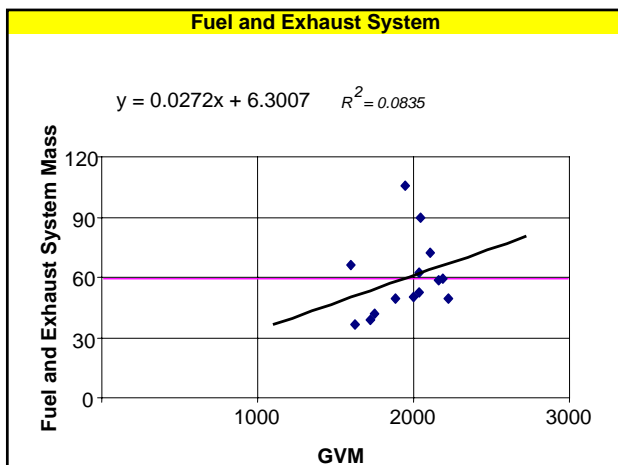
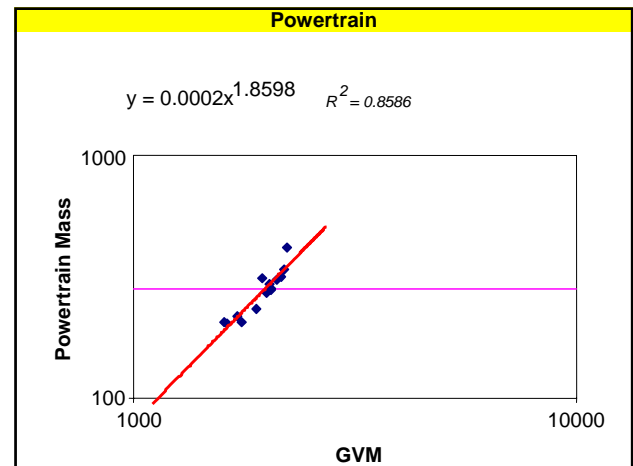
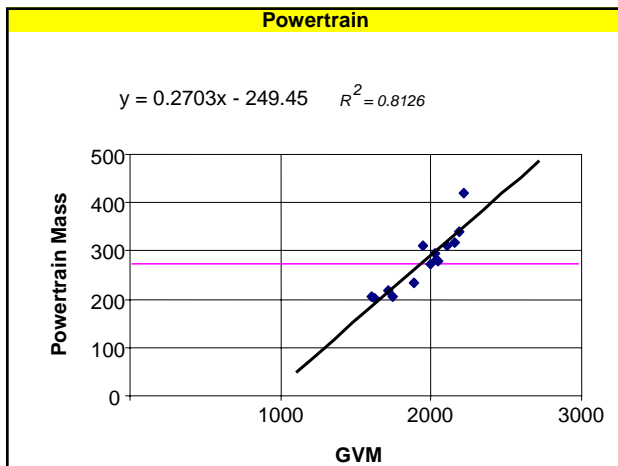
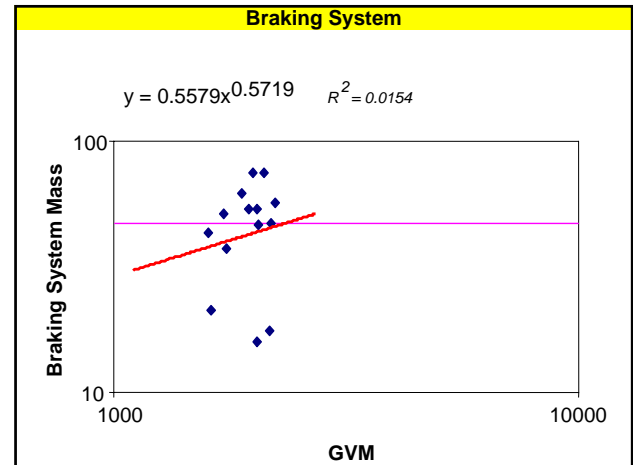
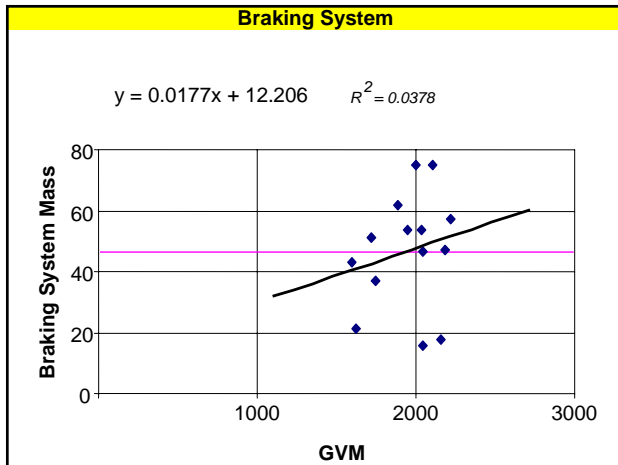


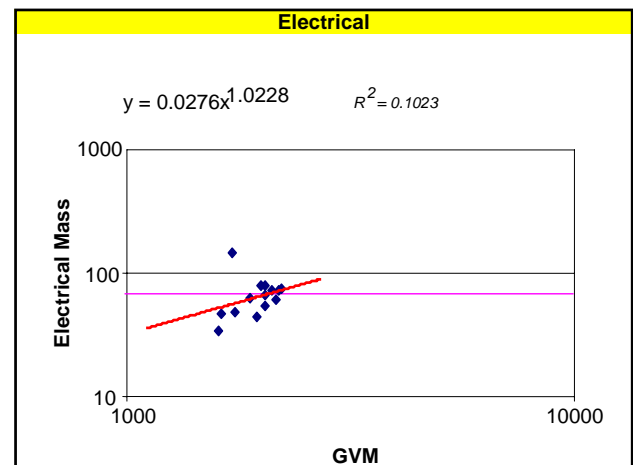
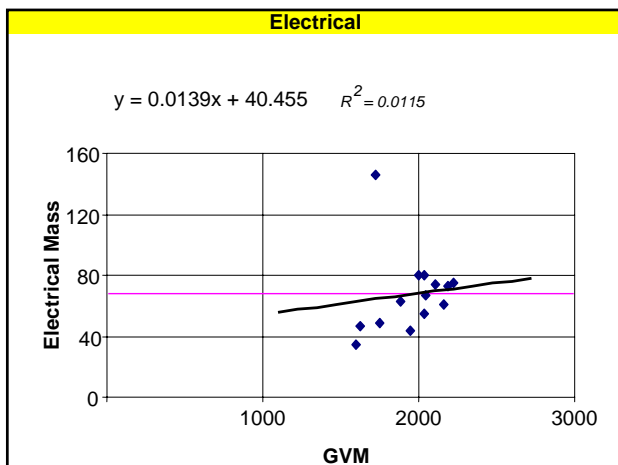
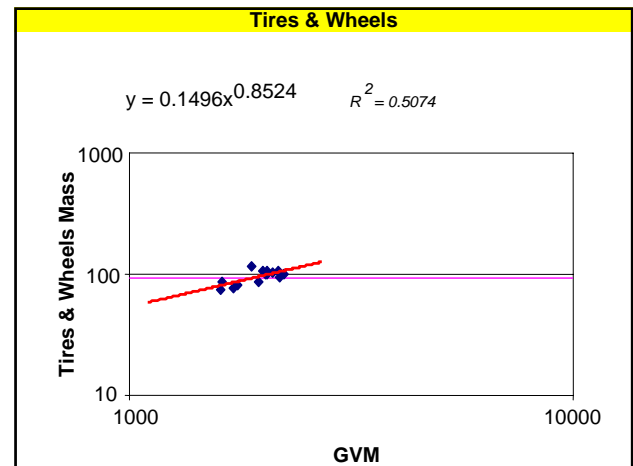
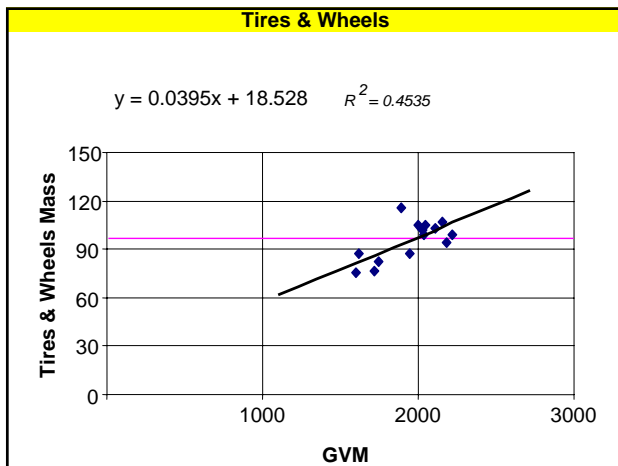
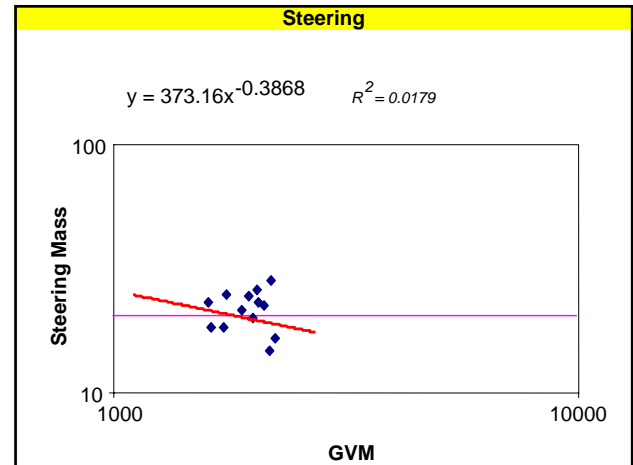
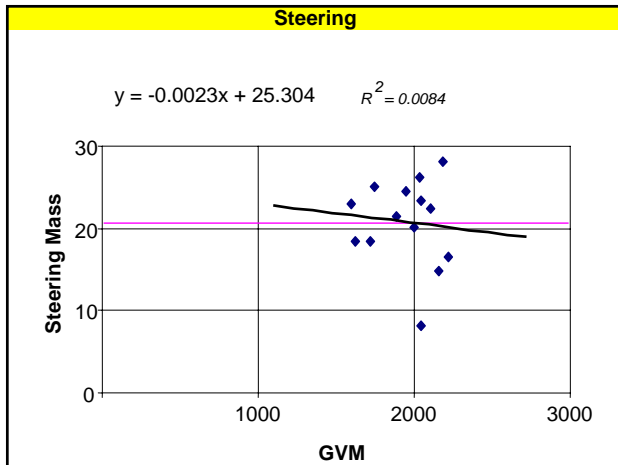


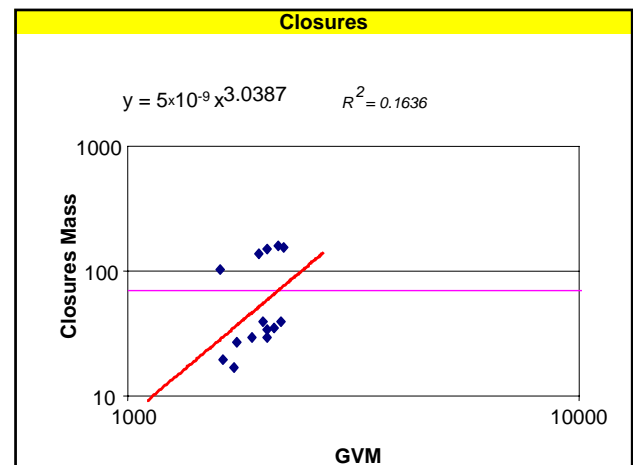
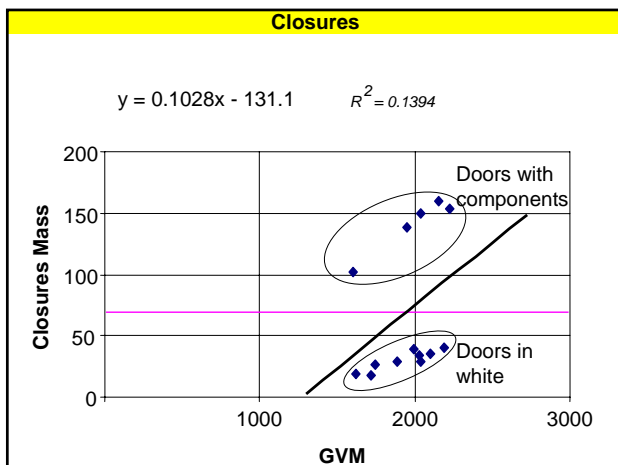
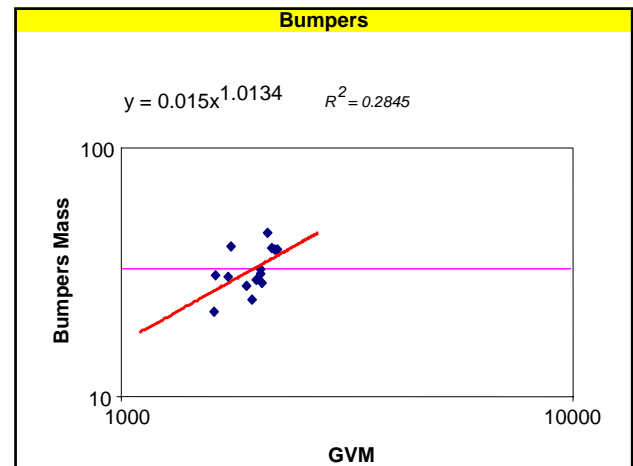
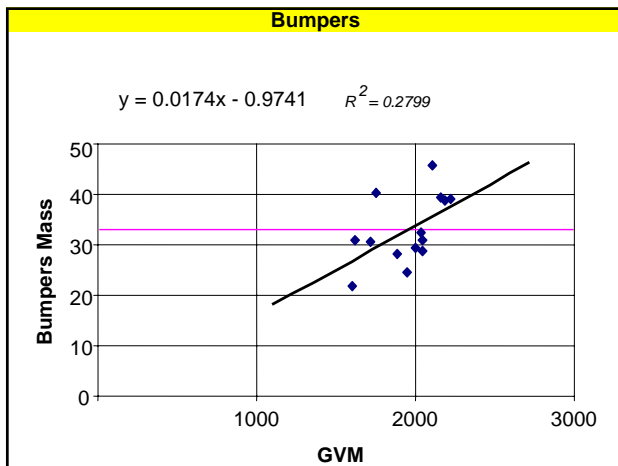
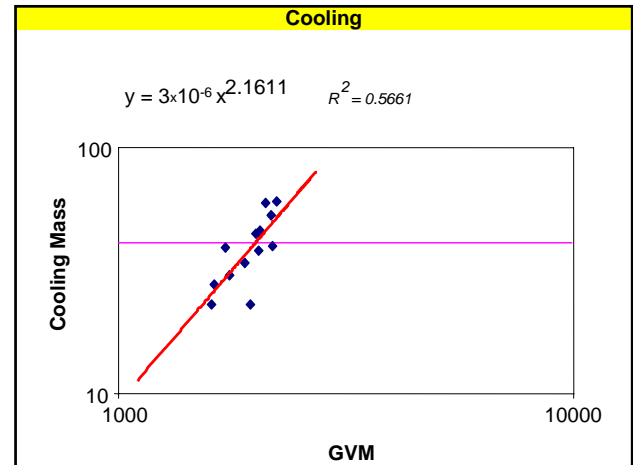
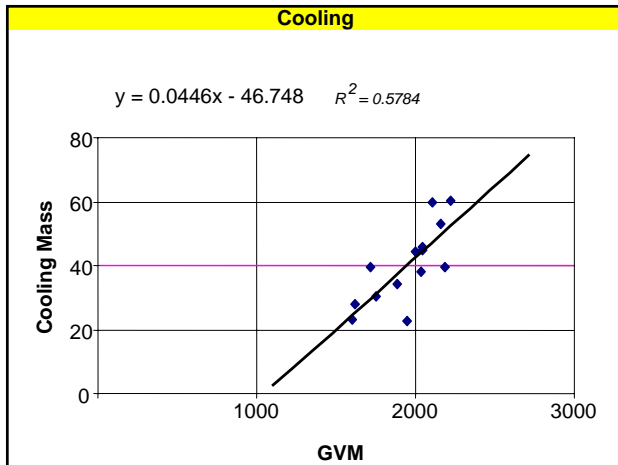


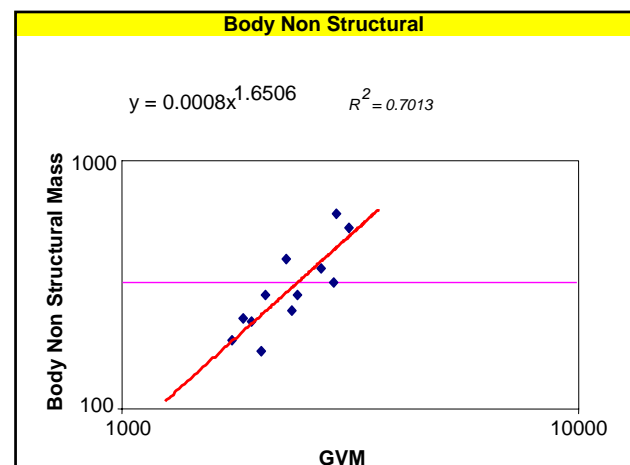
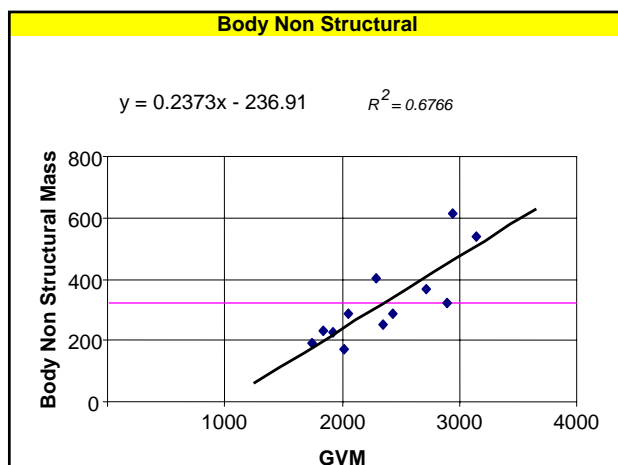
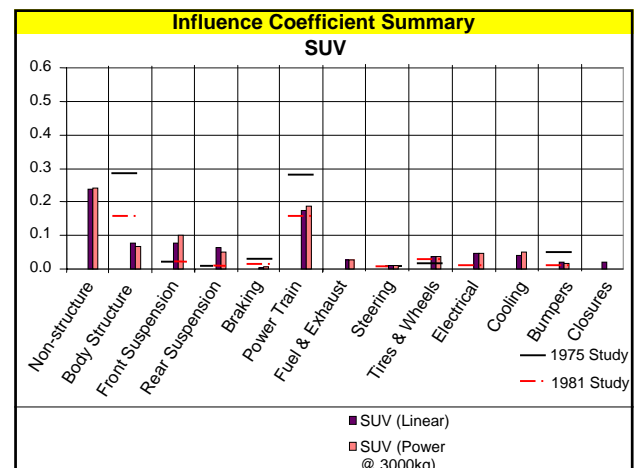
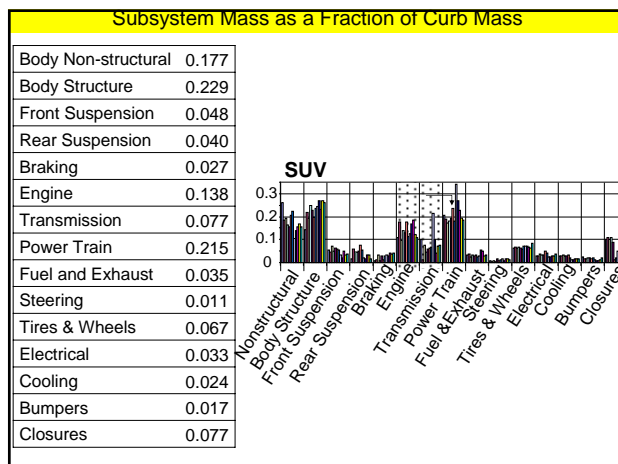
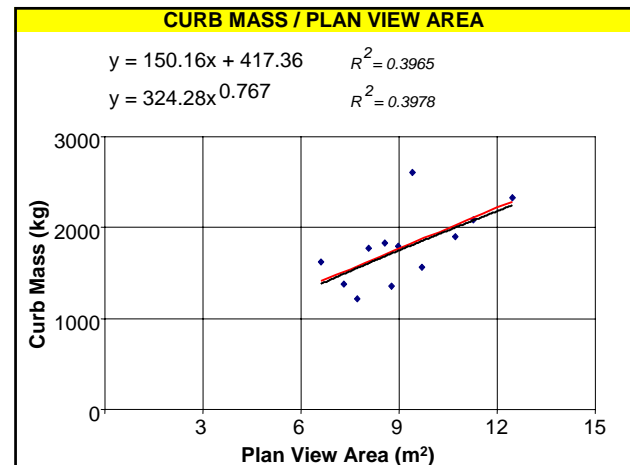
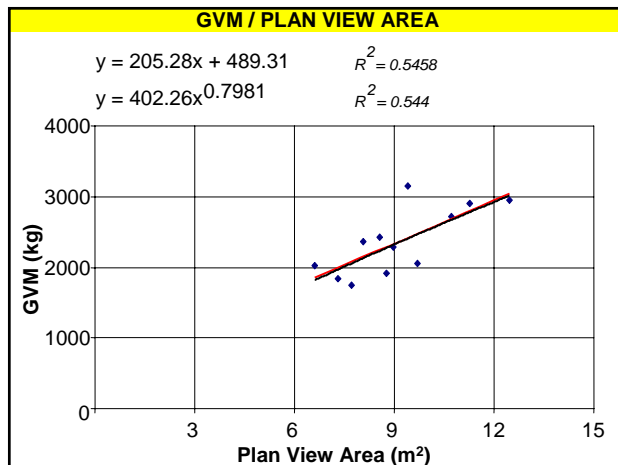


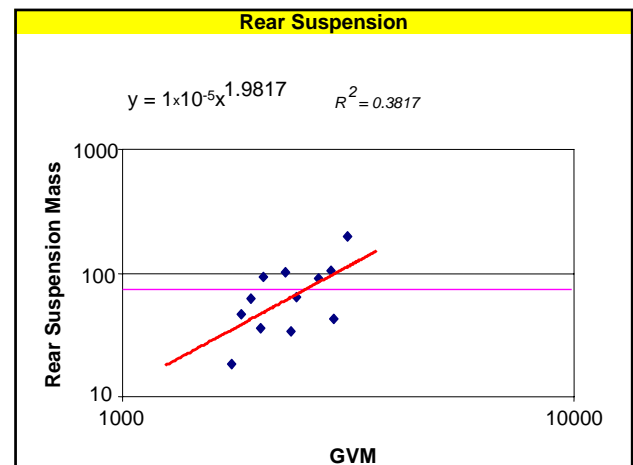
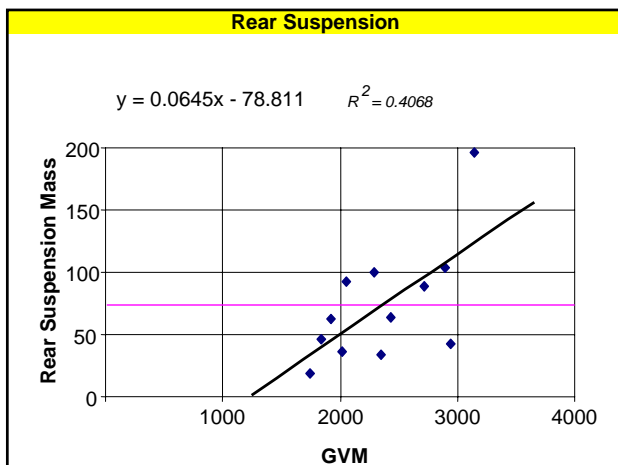
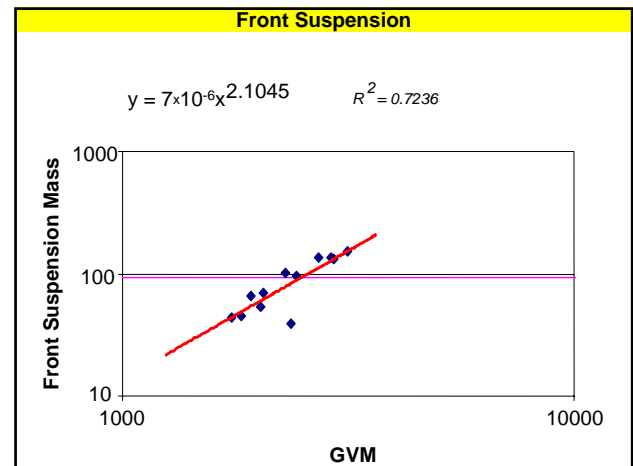
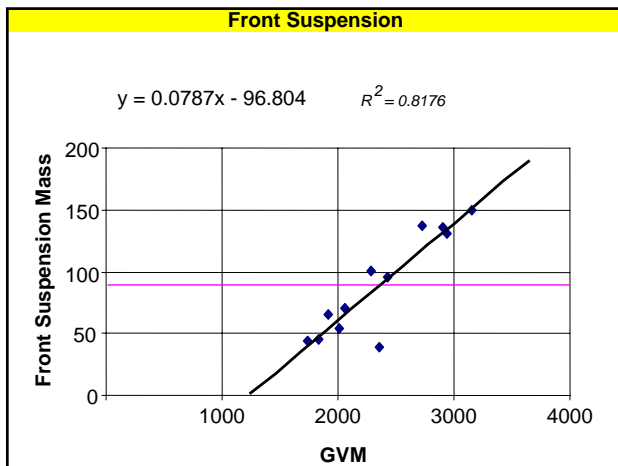
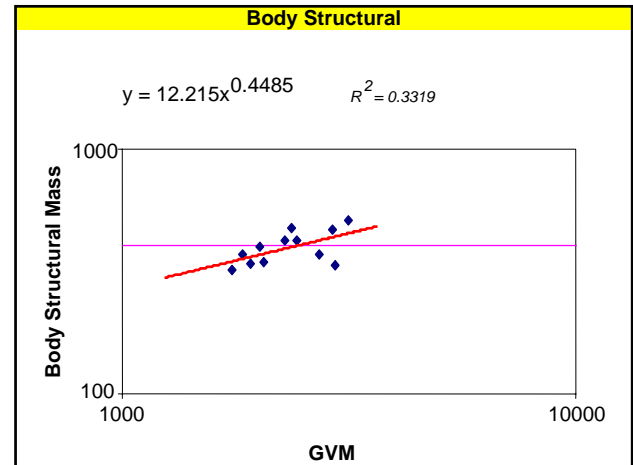
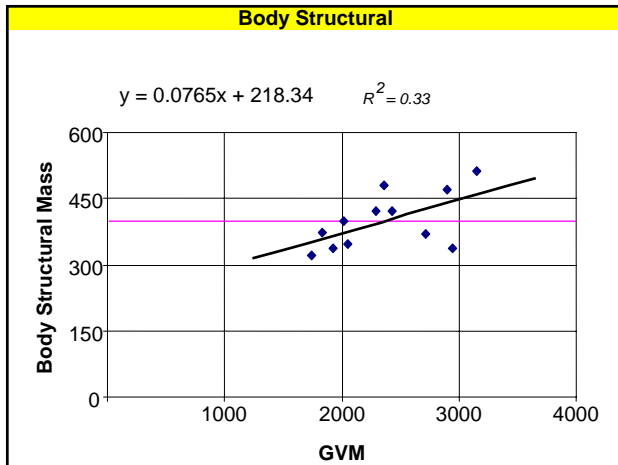


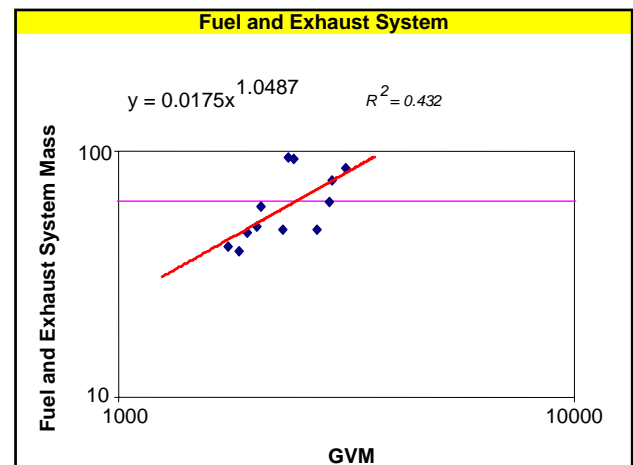
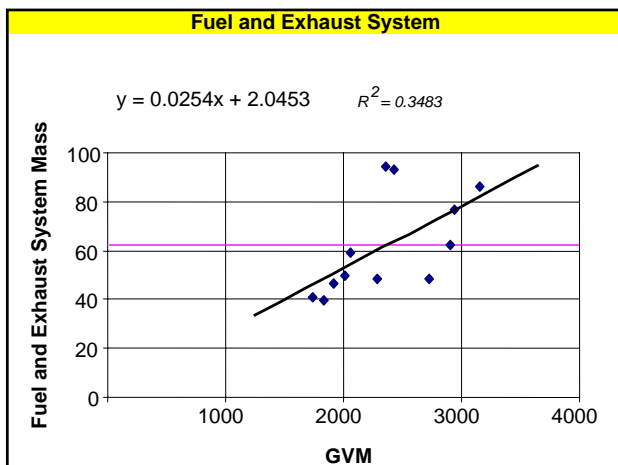
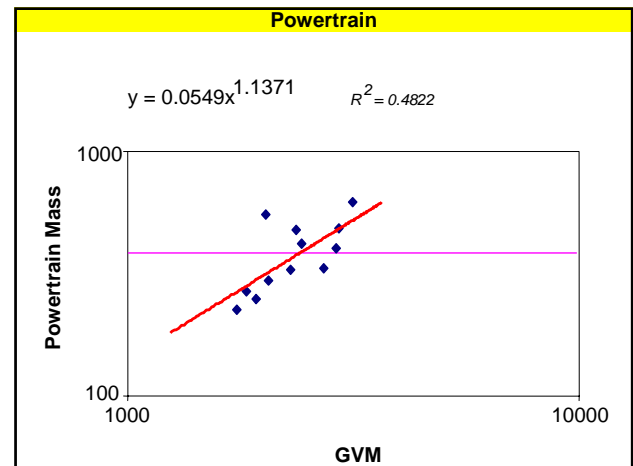
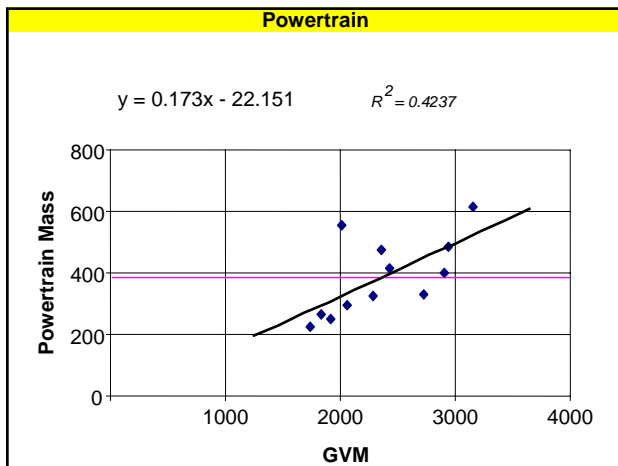
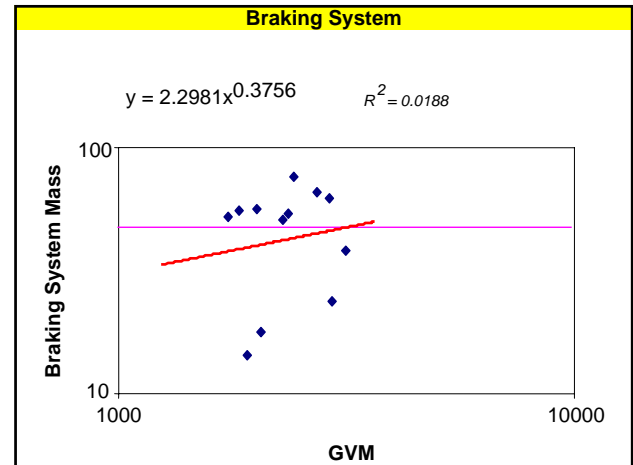
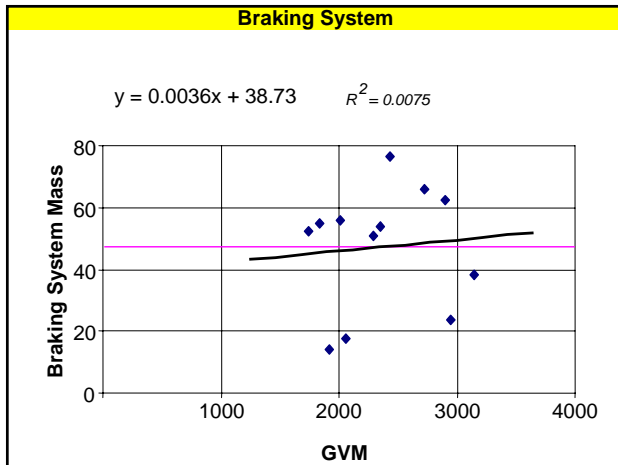


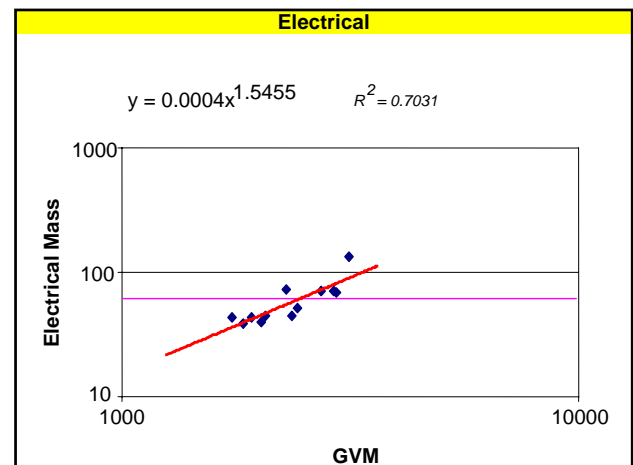
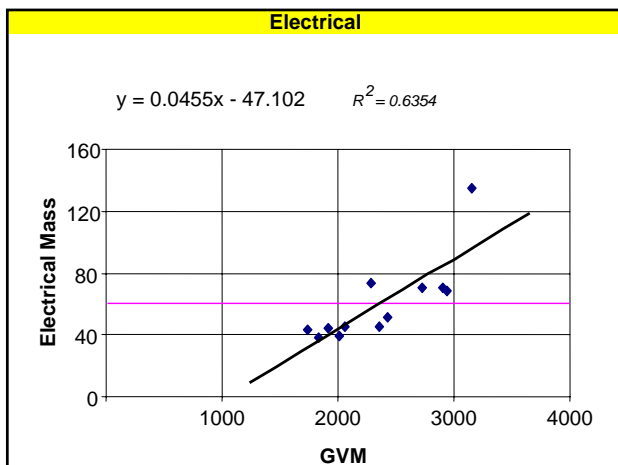
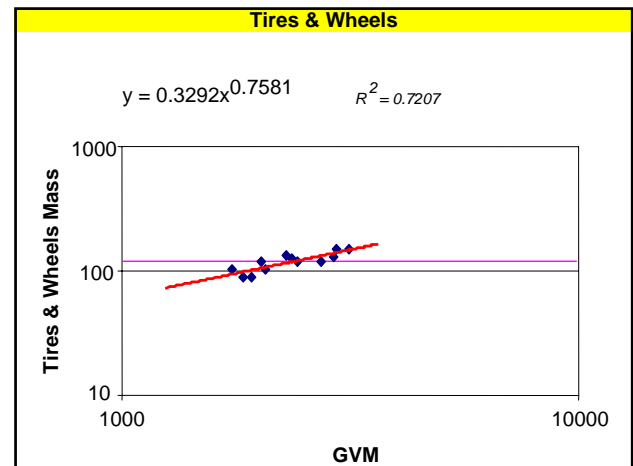
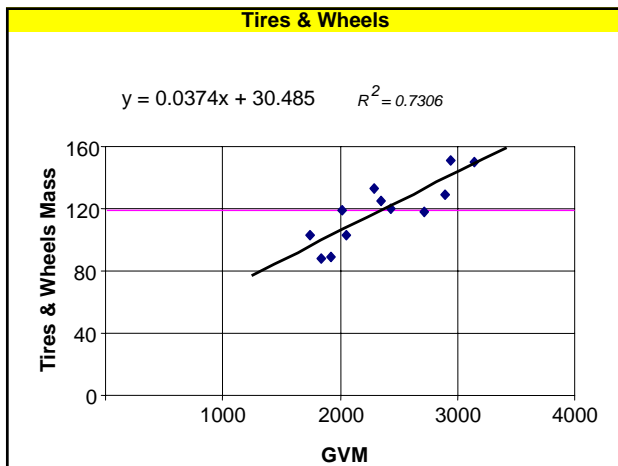
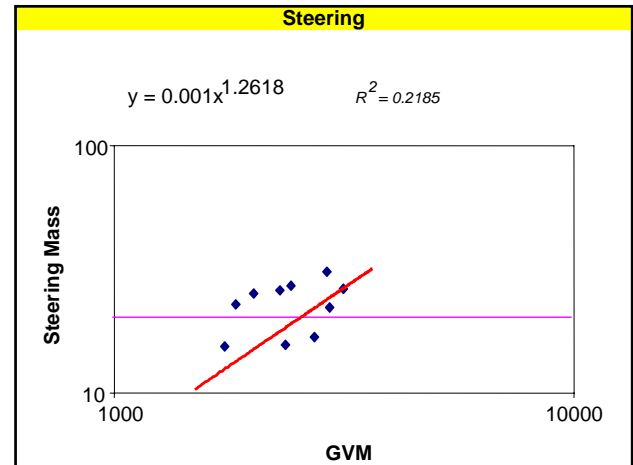
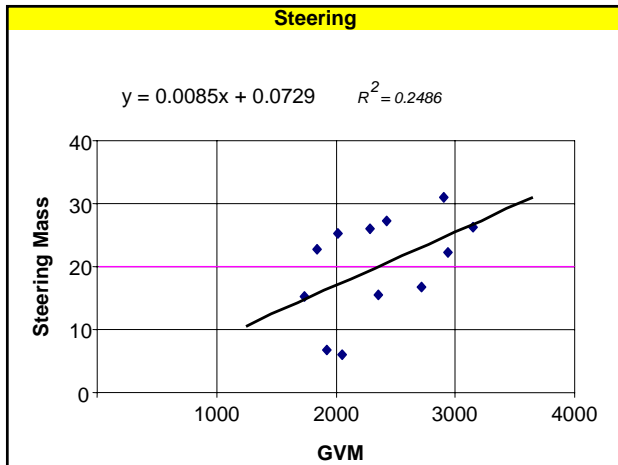


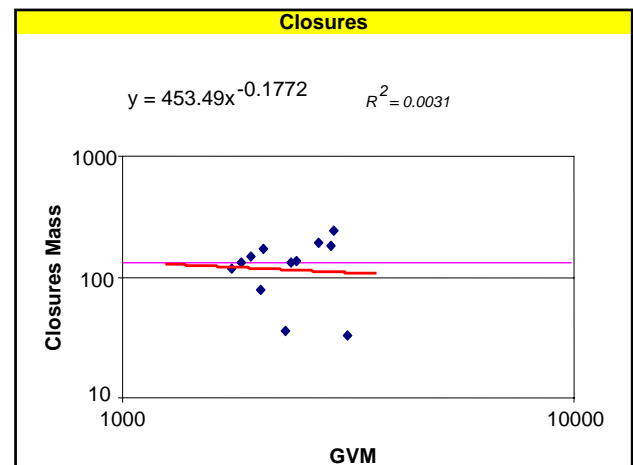
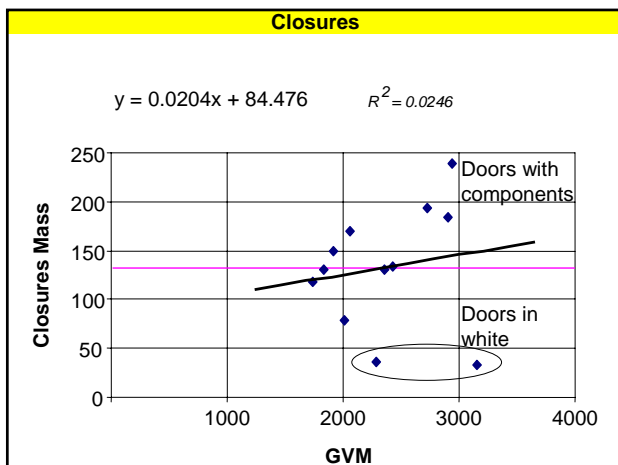
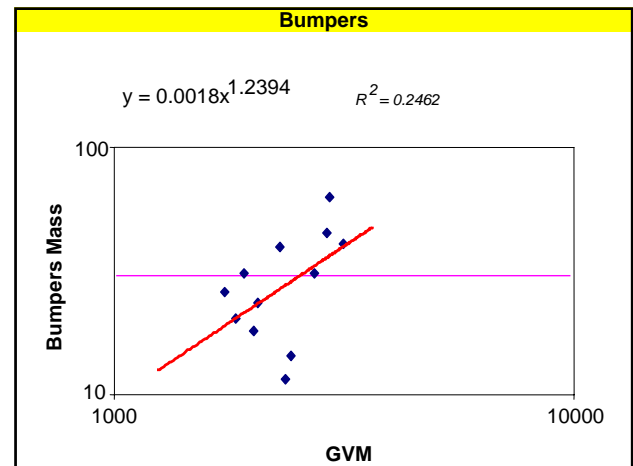
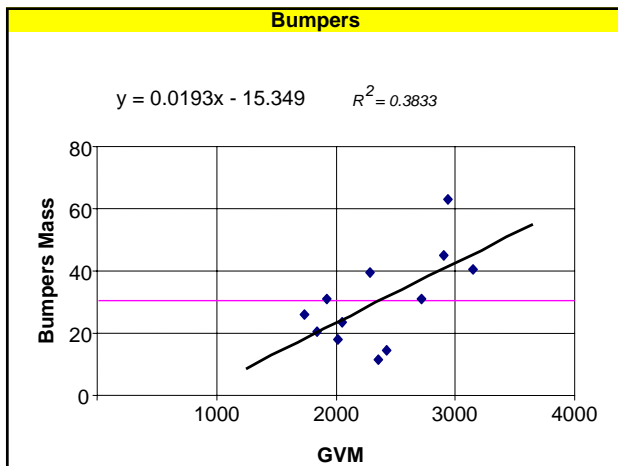
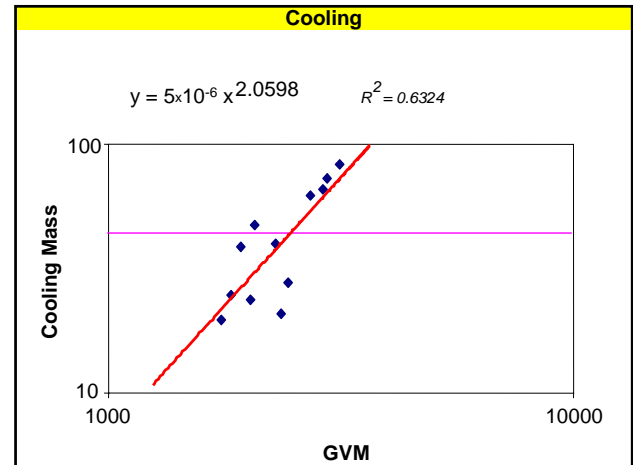
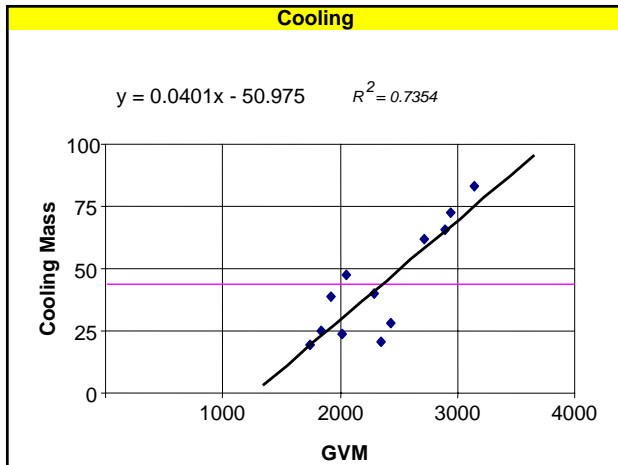


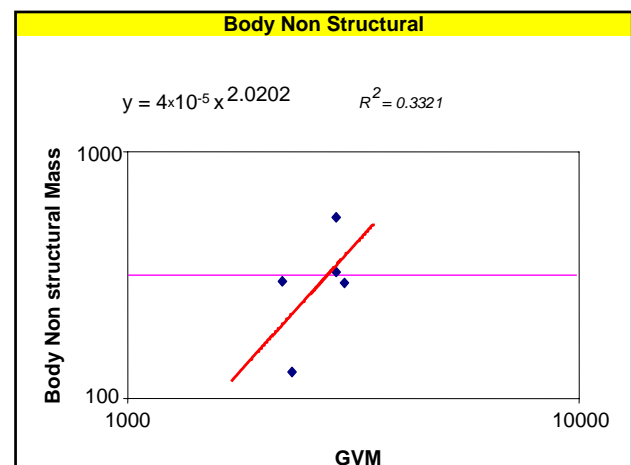
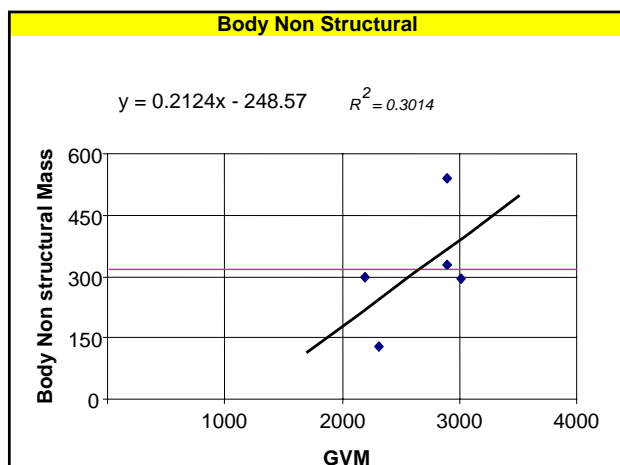
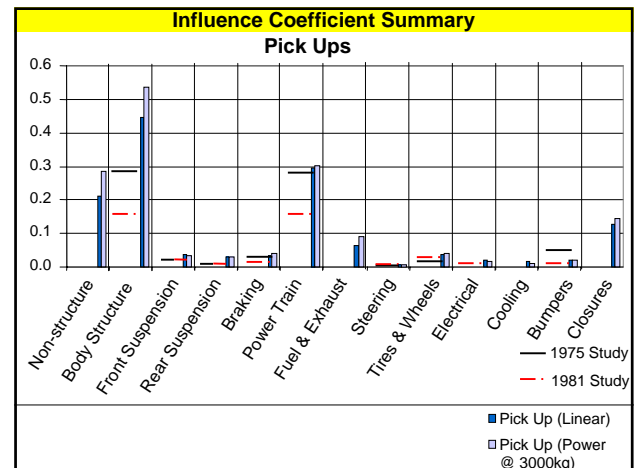
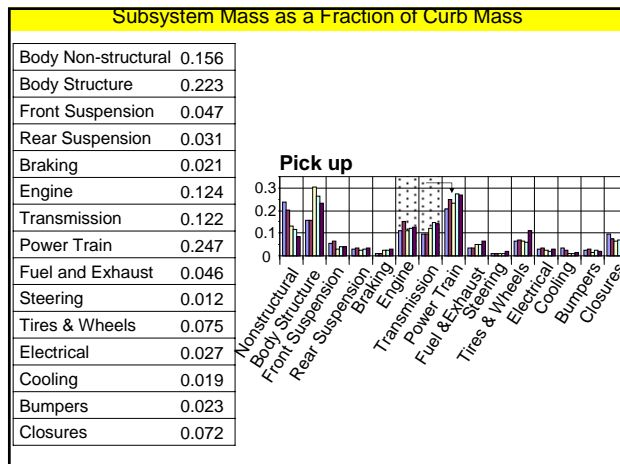
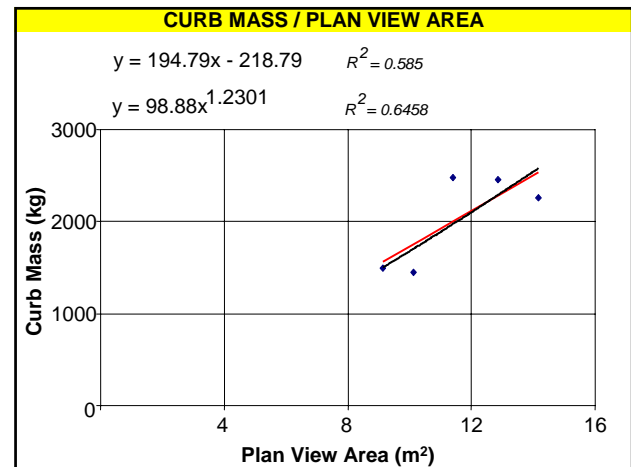
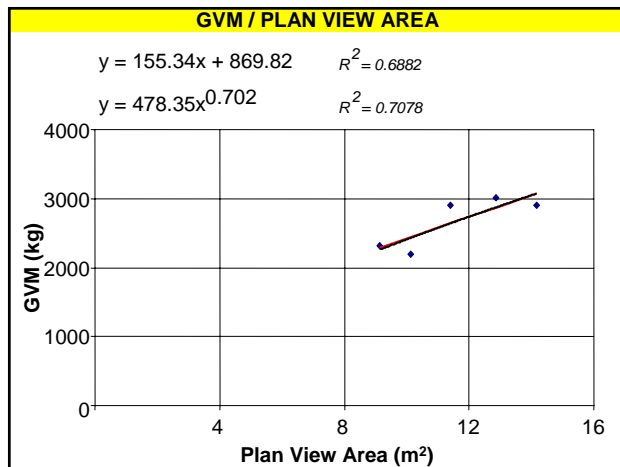


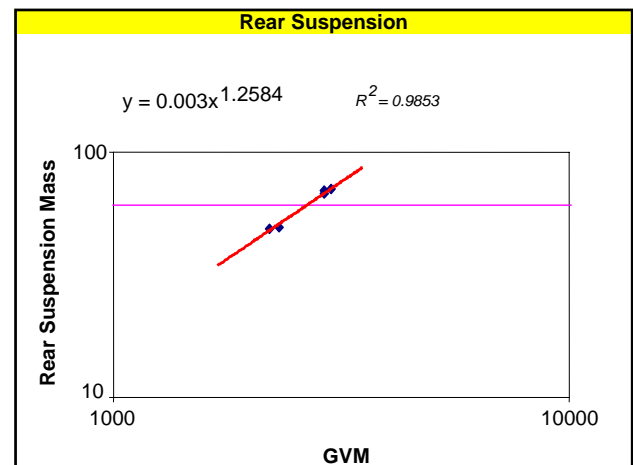
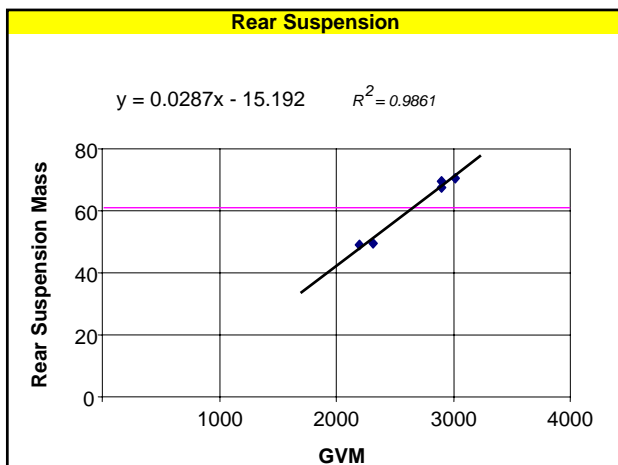
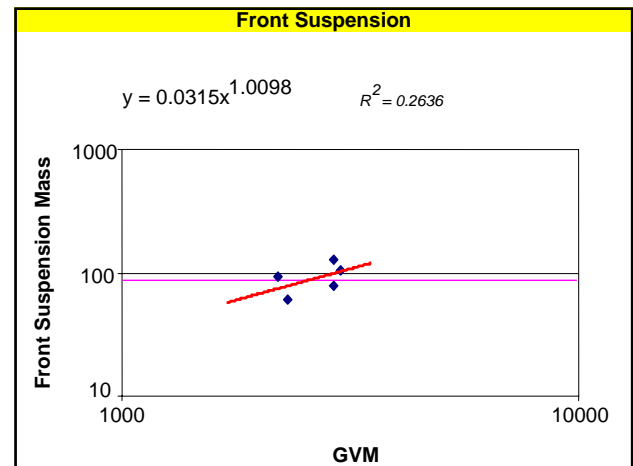
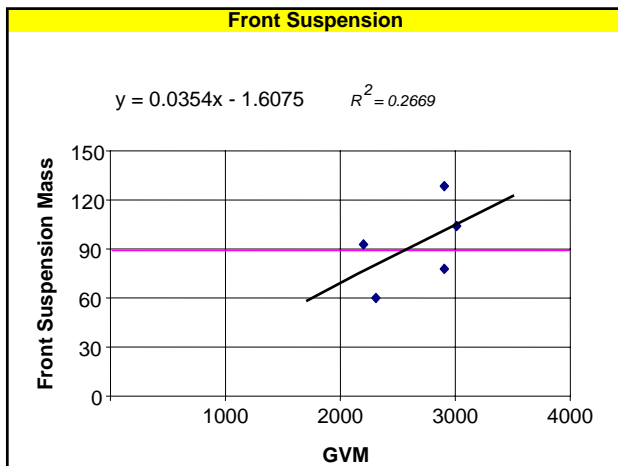
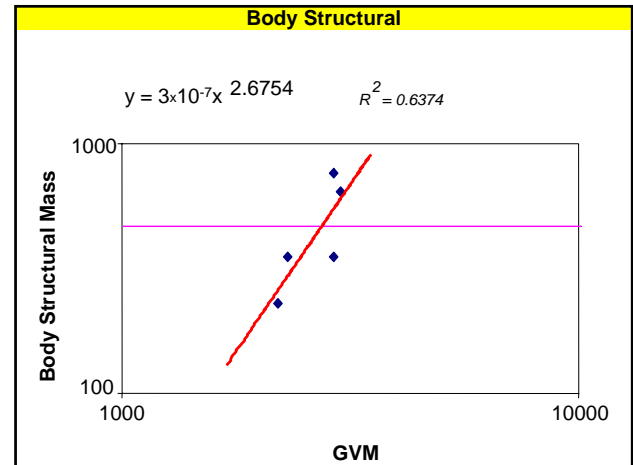
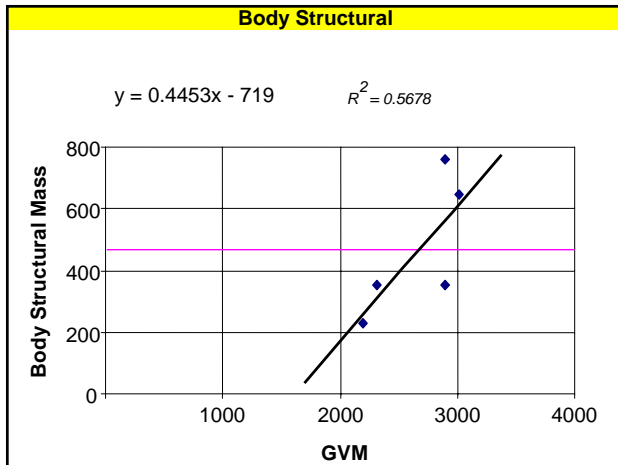


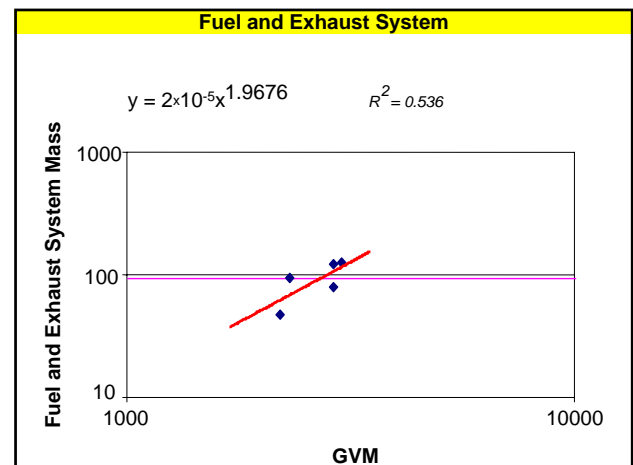
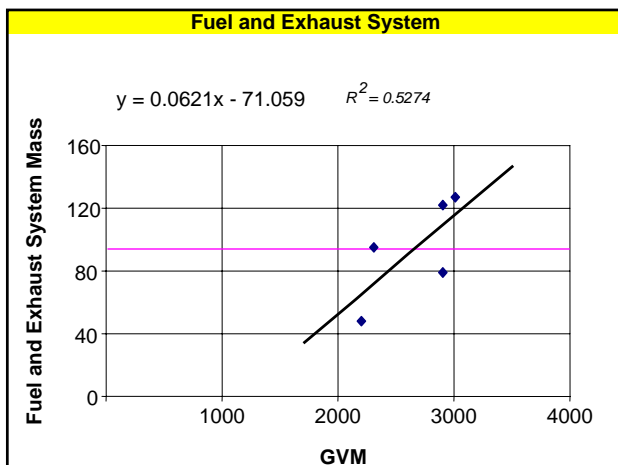
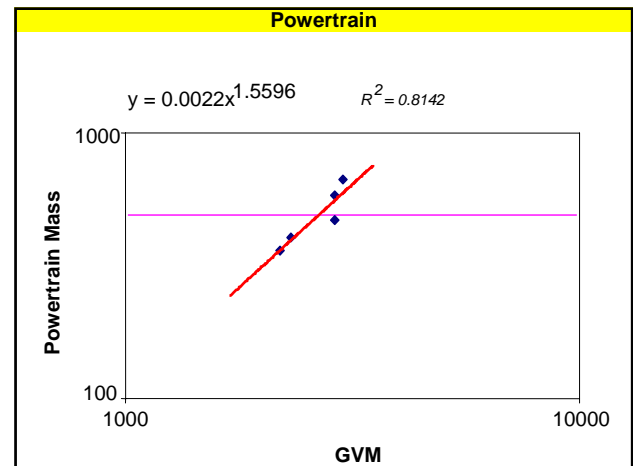
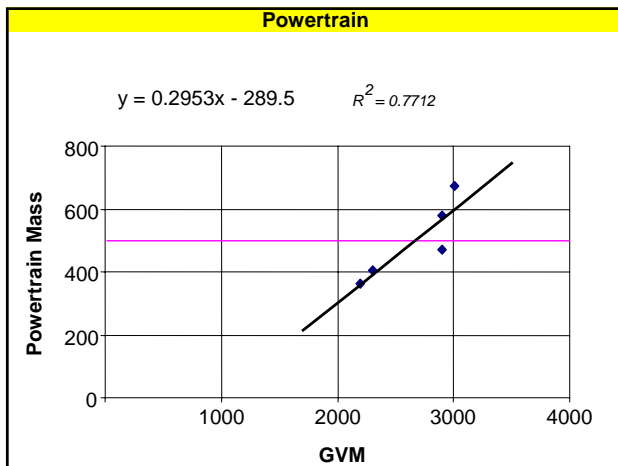
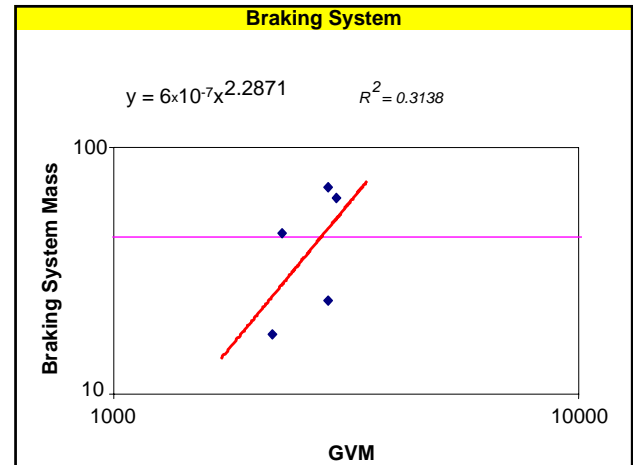
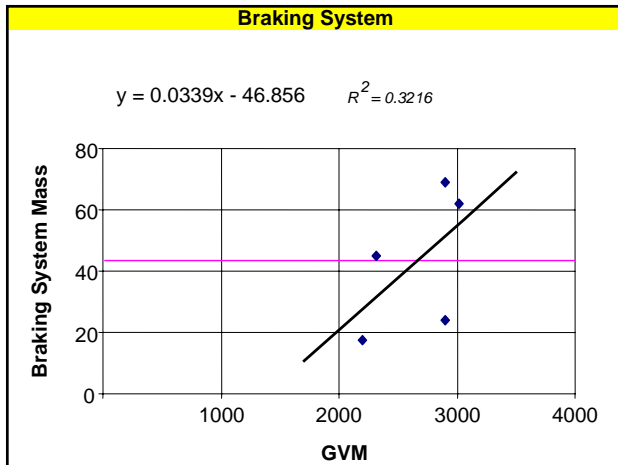


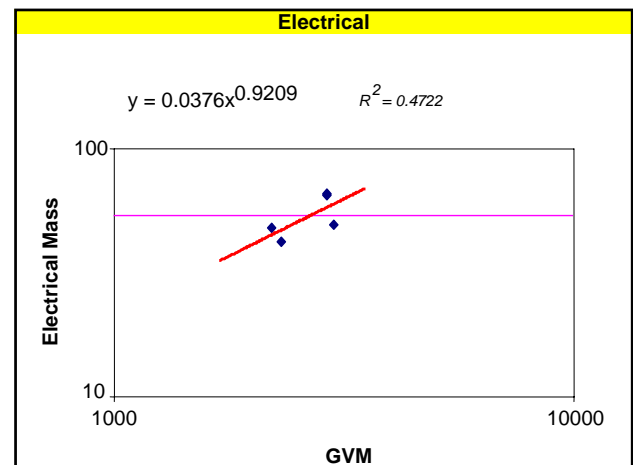
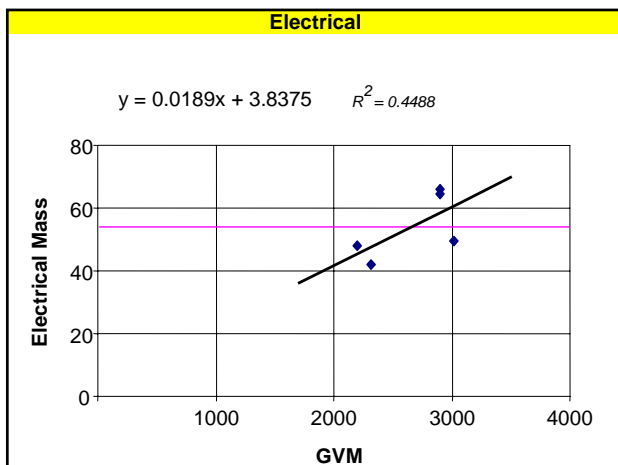
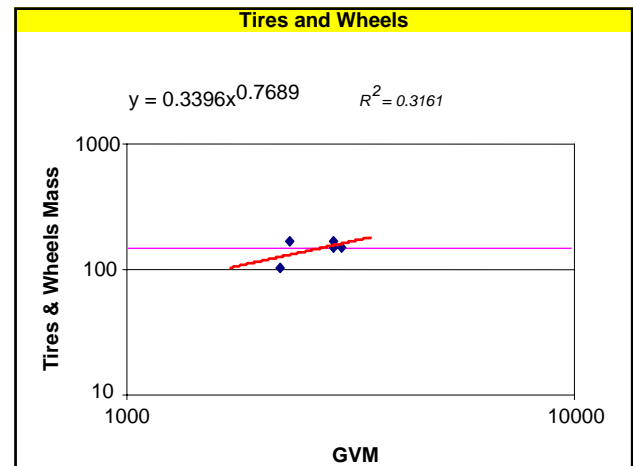
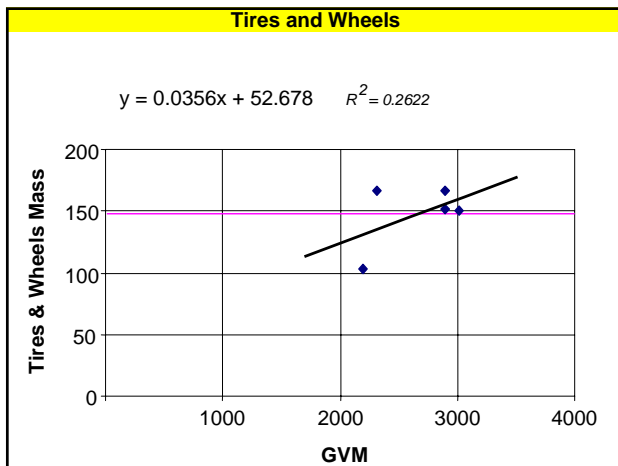
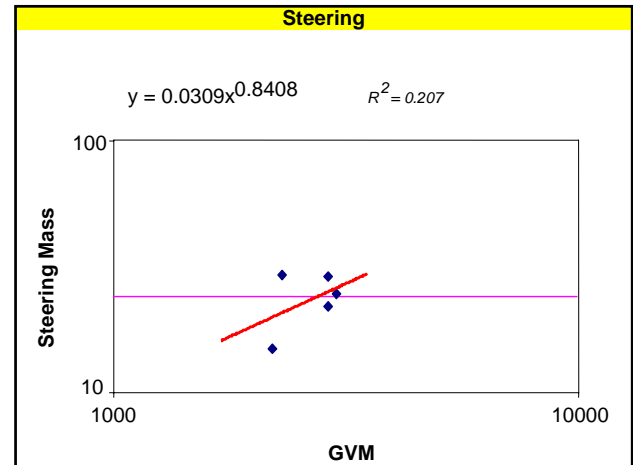
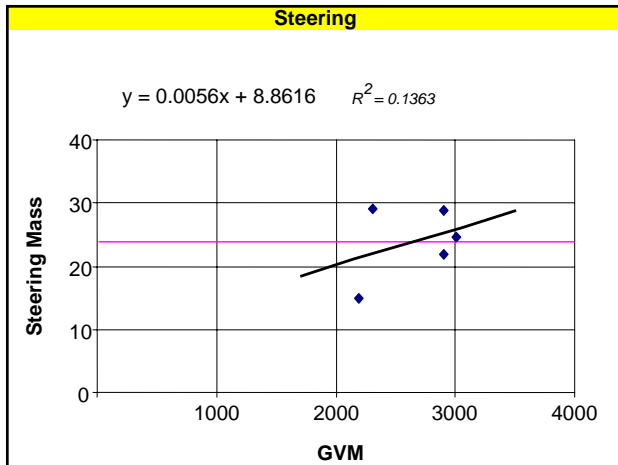


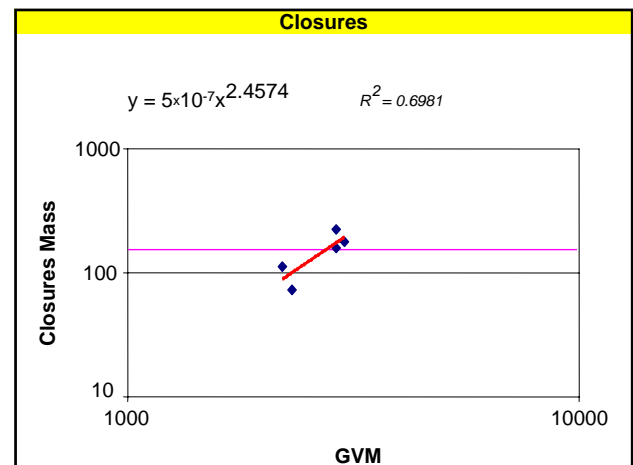
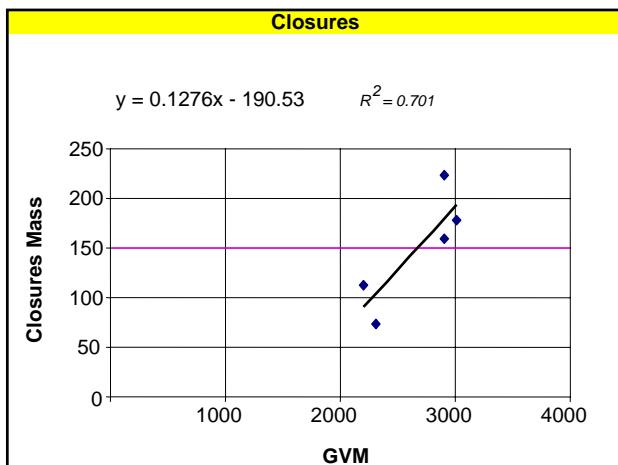
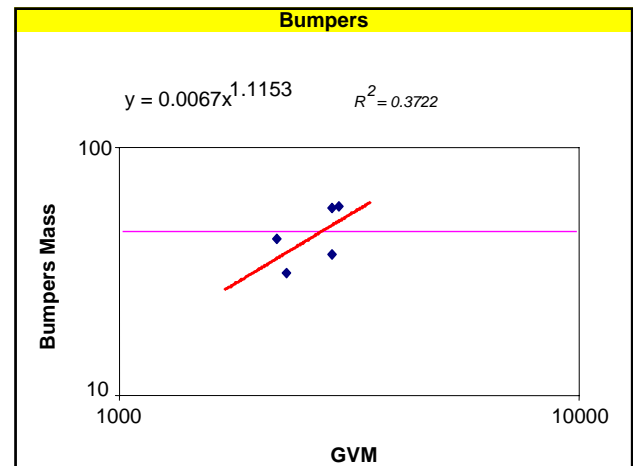
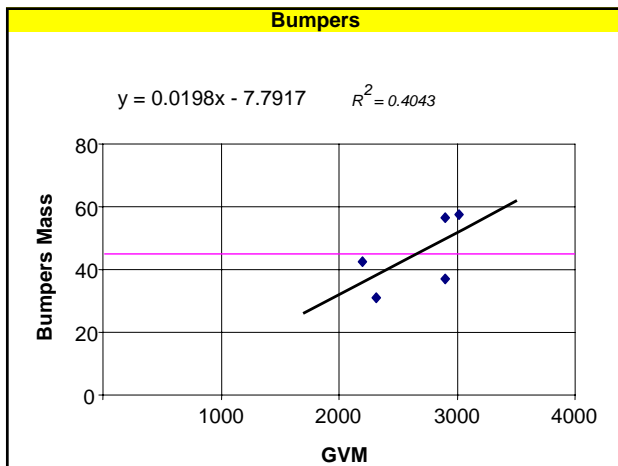
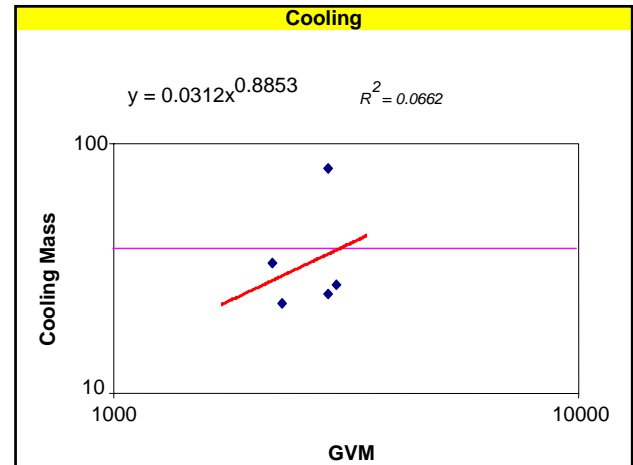
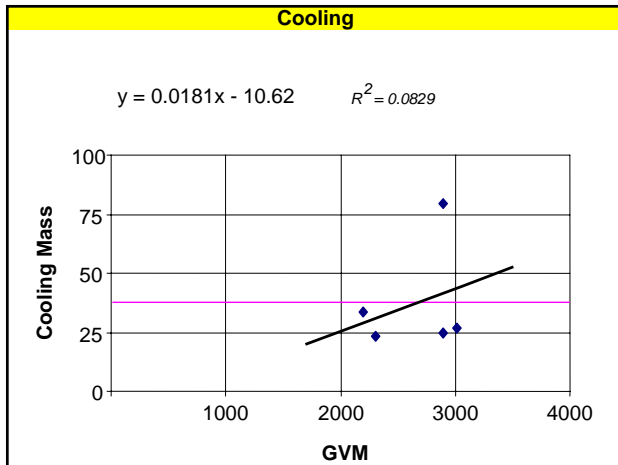












Influence Coefficients

Recommended subsystem coefficients
shown in bold with border

	Historical		All Vehicles				Sedan			SUV			Pick Up (Not significant as a group)		
	1975	1981	Linear	Power @ 1800 kg	Power @ 3000 kg	Power Equation $Y=CX^a$	Linear	Power @ 1800 kg	Power Equation $Y=CX^a$	Linear	Power @ 3000 kg	Power Equation $Y=CX^a$	Linear	Power @ 3000 kg	Power Equation $Y=CX^a$
Body Non-structural			0.145	0.131	0.127	$c=0.1988$ $a=0.9507$	0.142	0.153	$c=0.1465$ $a=1.0054$	0.237	0.242	$c=0.0008$ $a=1.6506$	0.212	0.285	$c=4E-05$ $a=2.0202$
Body Structure	0.294	0.166	0.176	0.161	0.154	$c=0.325$ $a=0.9176$	0.132	0.144	$c=0.8247$ $a=0.7928$	0.077	0.066	$c=12.215$ $a=0.4485$	0.445	0.537	$c=3E-07$ $a=2.6754$
Front Suspension	0.021	0.029	0.045	0.037	0.039	$c=0.0137$ $a=1.1176$	0.033	0.025	$c=0.631$ $a=0.622$	0.079	0.102	$c=7E-06$ $a=2.1045$	0.035	0.034	$c=0.0315$ $a=1.0098$
Rear Suspension	0.013	0.012	0.035	0.031	0.033	$c=0.0072$ $a=1.172$	0.089	0.058	$c=2E-07$ $a=2.5895$	0.065	0.051	$c=1E-05$ $a=1.9817$	0.029	0.030	$c=0.003$ $a=1.2584$
Braking System	0.038	0.016	0.008	0.010	0.007	$c=1.421$ $a=0.4422$	0.018	0.014	$c=0.5579$ $a=0.5719$	0.004	0.006	$c=2.2981$ $a=0.0188$	0.034	0.041	$c=6E-07$ $a=2.2871$
Powertrain	0.183	0.131	0.230	0.206	0.262	$c=0.004$ $a=1.4741$	0.270	0.200	$c=0.0002$ $a=1.8598$	0.173	0.187	$c=0.0549$ $a=1.1371$	0.295	0.303	$c=0.0022$ $a=1.5596$
Fuel and Exhaust			0.033	0.031	0.033	$c=0.0103$ $a=1.1318$	0.027	0.030	$c=0.0197$ $a=1.0525$	0.025	0.027	$c=0.0175$ $a=1.0487$	0.062	0.091	$c=2E-05$ $a=1.9676$
Steering	0.011	0.031	0.006	0.006	0.005	$c=0.1265$ $a=0.6583$	-0.002 (0)	-0.006 (0)	$c=373.16$ $a=-0.3959$	0.009	0.010	$c=0.001$ $a=1.2618$	0.006	0.007	$c=0.0309$ $a=0.8408$
Tires & Wheels	0.036	0.022	0.049	0.050	0.049	$c=0.058$ $a=0.9818$	0.040	0.043	$c=0.1496$ $a=0.8524$	0.037	0.036	$c=0.3292$ $a=0.7581$	0.036	0.041	$c=0.3396$ $a=0.7689$
Electrical		0.006	0.014	0.017	0.014	$c=0.7054$ $a=0.5751$	0.014	0.033	$c=0.0276$ $a=1.0228$	0.046	0.049	$c=0.0004$ $a=1.5455$	0.019	0.018	$c=0.0376$ $a=0.9209$
Cooling			0.021	0.016	0.016	$c=0.0225$ $a=0.9627$	0.045	0.032	$c=3E-06$ $a=2.1611$	0.040	0.050	$c=5E-06$ $a=2.0598$	0.018	0.011	$c=0.0312$ $a=0.8853$
Bumpers	0.048	0.006	0.013	0.010	0.008	$c=0.2118$ $a=0.6462$	0.017	0.017	$c=0.015$ $a=1.0134$	0.019	0.015	$c=0.0018$ $a=1.2394$	0.020	0.019	$c=0.0067$ $a=1.1153$
Closures			0.087	0.069	0.122	$c=8E-06$ $a=2.1099$	0.103	0.045	$c=5E-09$ $a=3.0387$	0.020	-0.006	$c=453.49$ $a=-0.1772$	0.128	0.144	$c=5E-07$ $a=2.4574$
Sum of recommended coefficients	0.644	0.413	0.562				0.599			0.463			0.900		

Selection of Influence Coefficients for Inclusion in Model

Inclusion Criteria	Regression Coefficient R^2					Physical link with vehicle mass	γ Magnitude	Included in prior work	Include
	All	Sedan	SUVs	Pick	Average				
Non structure	0.36	0.23	0.70	0.33	0.41*	Little	0.20	No	No
Body structure	0.47	0.69	0.33	0.64	0.53	Strong	0.15	Yes	Yes
Front	0.41	0.11	0.82	0.27	0.40	Strong	0.03	Yes	Yes
Rear	0.21	0.37	0.41	0.99	0.49	Strong	0.03	Yes	Yes
Braking	0.03	0.04	0.02	0.32	0.10	Strong	0.02	Yes	Yes
Powertrain	0.73	0.86	0.48	0.81	0.72	Strong	0.20	Yes	Yes
Fuel & exhaust	0.37	0.14	0.43	0.54	0.37	Little	0.03	No	No
Steering	0.14	0.02	0.25	0.21	0.15	Moderate	<0.01	Yes	(Yes)
Tire & wheels	0.74	0.51	0.73	0.32	0.57	Strong	0.04	Yes	Yes
Electrical	0.11	0.10	0.70	0.47	0.35	Little	0.02	Yes/No	No
Cooling	0.27	0.57	0.74	0.08	0.41	Moderate	0.02	No	No
Bumpers	0.21	0.28	0.38	0.40	0.32	Strong	0.01	Yes	Yes
Closures	0.30	0.16	0.02	0.70	0.30	Little	0.09	No	No

Key

0.00-0.25

0.25 -0.50

0.50-1.00

*note: High R^2 coefficient for Non-structure subsystem assumed to be due to co linearity of plan view area with GVM:
 $W_{\text{non structure}} \rightarrow (\text{interior space}) \rightarrow (\text{plan view area}) \rightarrow (\text{GVM})$



2000 Town Center, Suite 320
Southfield, Michigan 48075
Tel: 248.945.4777
www.a-sp.org

